The orbital decay of the semi-major axis of LARES and the LARASE contribution to SRL measurements for applications in the fields of space geodesy and geophysics

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G6.1 – Open Session on Geodesy, Vienna, Austria, 22 April 2016, 10:30-12:00
LARASE GOALS

- The LAser RAnged Satellite Experiment (LARASE) main goal is to provide accurate measurements for the gravitational interaction in the weak-field and slow-motion limit of General Relativity by means of a very precise laser tracking of geodetic satellites orbiting around the Earth.

- Beside the quality of the tracking observations, guaranteed by the powerful Satellite Laser Ranging (SLR) technique of the International Laser Ranging Service (ILRS), also the quality of the dynamical models implemented in the Precise Orbit Determination (POD) software plays a fundamental role in order to obtain precise and accurate measurements.

- The models have to account for the perturbations due to both gravitational and non-gravitational forces in such a way to reduce as better as possible the difference between the observed range, from the tracking, and the computed one, from the models.

- In particular, LARASE aims to improve the dynamical models of the current best laser-ranged satellites in order to perform a precise and accurate orbit determination, able to benefit also space geodesy and geophysics.
The accurate modeling of both gravitational and non-gravitational perturbations, coupled with a range accuracy approaching 1 cm, makes possible an orbit determination of comparable accuracy.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LARES</th>
<th>LAGEOS</th>
<th>LAGEOS II</th>
</tr>
</thead>
<tbody>
<tr>
<td>a [km]</td>
<td>7 828</td>
<td>12 270</td>
<td>12 163</td>
</tr>
<tr>
<td>e</td>
<td>0</td>
<td>0.004</td>
<td>0.014</td>
</tr>
<tr>
<td>I [deg]</td>
<td>69.5</td>
<td>109.9</td>
<td>52.7</td>
</tr>
<tr>
<td>R [cm]</td>
<td>18.2</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>M [kg]</td>
<td>386.8</td>
<td>406.9</td>
<td>405.4</td>
</tr>
<tr>
<td>A/M [m^2/kg]</td>
<td>2.69\cdot10^{-4}</td>
<td>6.94\cdot10^{-4}</td>
<td>6.97\cdot10^{-4}</td>
</tr>
</tbody>
</table>
PERTURBATIONS ON LAGEOS II & LARES

- Despite the smaller A/M ratio, the non-gravitational accelerations are not always smaller in magnitude for LARES with respect to LAGEOS II (or LAGEOS), due to the lower height (1450 vs. 5900 km) and the higher density of neutral atmosphere.

- Being 50 times larger on LARES than on the two LAGEOS, the accurate modeling of neutral atmosphere drag needs special attention, because it might mask the presence of smaller and subtler effects.

### Table: Non-Gravitational Accelerations

<table>
<thead>
<tr>
<th>Effect</th>
<th>Estimate</th>
<th>LAGEOS II</th>
<th>LARES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth’s monopole</td>
<td>( \frac{GM_{\oplus}}{r^2} )</td>
<td>2.69</td>
<td>6.51</td>
</tr>
<tr>
<td>Earth’s oblateness</td>
<td>( 3 \frac{GM_{\oplus}}{r^2} \left( \frac{R_{\oplus}}{r} \right)^2 \bar{C}_{2,0} )</td>
<td>(-1.1 \times 10^{-3})</td>
<td>(-6.4 \times 10^{-3})</td>
</tr>
<tr>
<td>Low-order geopotential harmonics</td>
<td>( 3 \frac{GM_{\oplus}}{r^2} \left( \frac{R_{\oplus}}{r} \right)^2 \bar{C}_{2,2} )</td>
<td>(5.4 \times 10^{-6})</td>
<td>(3.2 \times 10^{-5})</td>
</tr>
<tr>
<td>High-order geopotential harmonics</td>
<td>( 19 \frac{GM_{\oplus}}{r^2} \left( \frac{R_{\oplus}}{r} \right)^{18} \bar{C}_{18,18} )</td>
<td>(1.4 \times 10^{-12})</td>
<td>(4.6 \times 10^{-9})</td>
</tr>
<tr>
<td>Moon perturbation</td>
<td>( \frac{2GM_{\oplus}}{r^2} )</td>
<td>(2.2 \times 10^{-6})</td>
<td>(1.4 \times 10^{-6})</td>
</tr>
<tr>
<td>Sun perturbation</td>
<td>( \frac{2GM_{\odot}}{r^2} )</td>
<td>(9.6 \times 10^{-7})</td>
<td>(6.2 \times 10^{-7})</td>
</tr>
<tr>
<td>General relativistic correction</td>
<td>( \frac{GM_{\odot}GM_{\oplus}}{r^2 c^2} )</td>
<td>(9.8 \times 10^{-10})</td>
<td>(3.7 \times 10^{-9})</td>
</tr>
<tr>
<td>Atmospheric drag</td>
<td>( \frac{1}{2} C_D \frac{A}{M} \rho V^2 )</td>
<td>(-2.6 \times 10^{-13})</td>
<td>(-1.3 \times 10^{-11})</td>
</tr>
<tr>
<td>Solar radiation pressure</td>
<td>( C_R \frac{A \Phi_{\odot}}{M c} )</td>
<td>(3.2 \times 10^{-9})</td>
<td>(1.2 \times 10^{-9})</td>
</tr>
<tr>
<td>Albedo radiation pressure</td>
<td>( C_R \frac{A \Phi_{\odot}}{M c} \left( \frac{R_{\oplus}}{r} \right)^2 )</td>
<td>(3.5 \times 10^{-10})</td>
<td>(2.4 \times 10^{-10})</td>
</tr>
<tr>
<td>Thermal emission</td>
<td>( \frac{4 A \Phi_{\odot} \Delta T}{9 M c \Delta T_0} )</td>
<td>(2.8 \times 10^{-11})</td>
<td>not available</td>
</tr>
<tr>
<td>Dynamic solid tide</td>
<td>( 3k_2 \frac{GM_{\odot}}{\kappa} \left( \frac{R_{\oplus}}{\kappa} \right)^2 \frac{R_{\oplus}^3}{r^4} )</td>
<td>(3.7 \times 10^{-6})</td>
<td>(2.2 \times 10^{-5})</td>
</tr>
<tr>
<td>Dynamic ocean tide</td>
<td>( \sim 0.1 ) of the dynamic solid tide</td>
<td>(3.7 \times 10^{-7})</td>
<td>(2.2 \times 10^{-6})</td>
</tr>
</tbody>
</table>
OBSERVED ORBITAL DECAY OF LARES

- From a POD of LARES over a time span of about 3.7 years, we have been able to measure a mean orbital decay in the residuals of its semi-major axis of about $-1$ m per year, i.e. $-2.74$ mm per day.
- This POD has been obtained analyzing the LARES normal points with the GEODYN II (NASA/GSFC) software.
- Neither the neutral and charged atmosphere drag, nor the thermal effects, have been included in the dynamical models.
- The corresponding unmodeled mean transversal acceleration of about $-1.444 \times 10^{-11}$ m/s$^2$ then includes all the effects of the perturbations not taken into account in the POD and eventually giving a secular and/or long-period contribution to the transversal acceleration component.
- The first line of attack was therefore the accurate modeling of neutral atmosphere drag, in order to evaluate how much of the unaccounted for acceleration can be explained by current thermospheric density models.
OBSERVED ORBITAL DECAY OF LARES

\[ y = -0.027345x + 0.10064 \]
DETAILED DRAG MODELING

- A modified version of the SATRAP tool, developed at ISTI/CNR, was used to compute the neutral drag acceleration acting on LARES, as a function of time, taking into account the real evolution of solar and geomagnetic activities and the observed secular semi-major axis decay.

- The following thermospheric density models were used within SATRAP to compute the components of the neutral drag acceleration in the reference system R (Radial), S (Transverse) and W (Out-of-Plane): JR-71, MSIS-86, MSISE-90, NRLMSISE-00 and GOST-2004.

- The analysis covered the first 3.7 years of LARES in orbit and the drag coefficient $C_D$ was adjusted, for each atmospheric density model, in order to reproduce the average decay of the semi-major axis by $-0.9988$ m/year, obtained through the analysis of the residuals of the GEODYN II precise orbit determination.
TRANSVERSE ACCELERATION COMPARED WITH SOLAR AND GEOMAGNETIC ACTIVITY
DRAG COMPONENTS WITH NRLMSISE-00

![Graph showing drag components with NRLMSISE-00 model](image)
SUMMARY OF THE RESULTS FOR LARES

For each thermospheric density model used in the analysis, the following mean adjusted drag coefficients were obtained, in order to reproduce the observed semi-major axis decay of LARES over the first 3.7 years of flight:

- JR-71 \(\langle C_D \rangle = 3.95\)
- MSIS-86 \(\langle C_D \rangle = 3.71\)
- MSISE-90 \(\langle C_D \rangle = 3.73\)
- NRLMSISE-00 \(\langle C_D \rangle = 3.78\)
- GOST-2004 \(\langle C_D \rangle = 4.21\)

The average drag coefficient among the 5 models was 3.88, with a maximum discrepancy of 8.6%, but MSIS-86, MSISE-90 and NRLMSISE-00 have a common heritage and are very similar.

Taking the average between JR-71, NRLMSISE-00 and GOST-2004, the mean drag coefficient resulted to be 3.98, with a maximum discrepancy of 5.8%.

The differences are well below the intrinsic uncertainties of the models, around 15% (or more).
INDEPENDENT CHECK WITH AJISAI

- A completely independent check of neutral atmosphere drag at the LARES altitude was possible thanks to another passive spherical satellite launched by Japan for geodetic research on 12 August 1986: AJISAI.

- With a diameter of 215 cm and a mass of 685 kg, the hollow sphere has an area-to-mass ratio $A/M = 5.30 \times 10^{-3} \text{ m}^2/\text{kg}$, i.e. 19.70 times that of LARES, being therefore much more sensitive to non-gravitational perturbations, in particular atmospheric drag.

- The nearly circular orbit has a semi-major axis of 7866.5 km, a mean geodetic altitude of 1494 km and an inclination of 50.0 deg.

- AJISAI is therefore 40 km higher than LARES.
AJISAI ORBITAL DECAY

- The orbital decay of AJISAI due to neutral atmosphere drag is known since its launch and has been used over the years to check the predictions of diverse atmospheric models under various conditions of solar and geomagnetic activity.

- During the approximately 3.7 years previously discussed for LARES, the average secular decrease of the semi-major axis was $-14.035$ m/year.

- Therefore, the magnitude of the observed secular semi-major axis decay was about $14.05$ times larger than for LARES.

- In order to reproduce the observed mean secular decrease of the semi-major axis, the needed average transverse component of the drag acceleration was $-2.012 \times 10^{-10}$ m/s$^2$.

- Factoring in the differences between average drag coefficients, area-to-mass ratios, atmospheric densities and squared relative velocities, it was straightforward to find the correct ratio ($\approx 14$) between the average secular transverse accelerations acting, respectively, on AJISAI and LARES, just assuming that one of these is explained by neutral atmosphere drag.
DRAG COMPONENTS WITH NRLMSISE-00

![Graph showing drag components](image)
SUMMARY OF THE RESULTS FOR AJISAI

- For each thermospheric density model used in the analysis, the following mean adjusted drag coefficients were obtained, in order to reproduce the observed semi-major axis decay of AJISAI over the 3.7 years considered:
  - JR-71 \( \langle C_D \rangle = 3.45 \)
  - NRLMSISE-00 \( \langle C_D \rangle = 3.22 \)
  - GOST-2004 \( \langle C_D \rangle = 3.42 \)

- The results of MSIS-86 and MSISE-90 are not shown again, being practically coincident with those of NRLMSISE-00

- Taking the average between JR-71, NRLMSISE-00 and GOST-2004, the mean drag coefficient resulted to be 3.36, with a maximum discrepancy of 4.2%

- Again, the differences are well below the intrinsic uncertainties of the models, around 15% (or more)

- Neutral atmosphere drag alone is able to account for the observed semi-major axis decay of AJISAI
CONCLUSIONS

- The results outlined strongly support the conclusion that most of the observed secular semi-major axis decay of LARES is due to neutral atmosphere drag.

- This conclusion is fully consistent with the predictions, uncertainties and range of applicability of some of the best thermospheric density models available and used by the orbital dynamics community.

- It is further strengthened by the totally independent results obtained with AJISAI, a spherical satellite orbiting at a similar altitude, but with quite different construction and surface properties, leading to a different response to non-gravitational perturbations.

- Contrary to what is happening in the case of LAGEOS and LAGEOS II, where neutral atmosphere drag accounts for less than 10% of the observed semi-major axis decay ($\approx -$0.2 m/year), for LARES it is a major player among non-gravitational perturbations and its secular, long-term and short-term signatures must be investigated and modeled in detail, in order to reliably detect and characterize other comparable, or smaller (depending on the RSW component), perturbing accelerations.
ONGOING WORK

- The work carried out on neutral atmosphere drag was just one of several aspects addressed in the framework of LARASE to deeply understand and evaluate all error sources affecting the primary and secondary goals of the experiment.

- It made possible to check and validate independently the conditions of applicability of the atmospheric density models implemented in GEODYN II.

- A detailed signature analysis is ongoing to characterize the various models, for instance the Russian GOST-2004 vs. the American JR-71 and NRLMSISE-00.

- Due to the absolute prevalence of neutral drag on LARES, this work is very important for the reliable identification and characterization of smaller non-gravitational perturbations, easily masked by the large thermospheric drag signal.

- All taken into account, an along-track unmodeled acceleration with a mean value of $-2.1 \times 10^{-13}$ m/s$^2$ (i.e. less than 1.5% of neutral atmosphere drag) was identified in the POD residuals of GEODYN II, probably attributable to thermal drag.