SOME REGULARITIES IN THE HADRON MASS SPECTRUM

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Abstract

An empirical rule is defined, by which the mass values of the lowest-lying strange and charmed hadrons are obtained from the mass values of the products of certain specific decays of these hadrons.
1. Introduction

In the past, as we can observe, empirical evidence for laws or relationships has contributed to theoretical developments in many areas of physics, even if, sometimes, over a long period of time. Usually, the search for an empirical evidence is encouraged only at the very beginning of research activity in a given field. On the contrary, when phenomenological models or theories start being developed, empirical laws that do not immediately integrate with the body of accumulated knowledge may be seen as a disturbing entry.

With the aim of finding "simple" regularities at the elementary level of matter, I have explored the possibility that the mass of a decaying hadron may be obtained through an empirical rule from the masses of the particles produced in the decay concerned. Surprisingly enough, such a rule can actually be defined with a degree of validity which is sufficiently high to support the conjecture that it may be not accidental.

2. Definition of an Empirical Rule

Let us consider the following hadron states: the lightest strange meson, $K(496)$; the four lightest strange baryons, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$, and $\Sigma(1385)$; the two other strange baryons of the decuplet, $\Xi(1530)$ and $\Omega(1672)$; and the lightest charmed baryon, $\Lambda_c^+(2282)$. For each of these states, let us then consider all those decays into hadrons (and, possibly, $\gamma$) which are not absolutely forbidden, and select only one of these
decays according to the following criterion. For each decay, write, first, the two sets of quarks and antiquarks which constitute the decaying state, on the one side, and the produced states, on the other side. Then, cancel, in the order, i) The pairs of quark-antiquark (i.e. $u\bar{u}$ or $d\bar{d}$) inside the set of the decay products; and ii) the equal quarks or antiquarks which belong to both sets. Call $J$ the maximum number of distinct quarks and antiquarks survived in either set. Finally, restrict our attention to that (those) decay(s) for which $J$ is minimum under the additional condition that the hadron states produced are $J+1$. If more than one decay meet the above requirements, then select only the one for which j) the total number of the above-called survived quarks and antiquarks (not necessarily distinct) is greater; and/or jj) the produced states, as a whole, have a greater mass; and/or jjj) the produced states are not the same as in one of the selected decays for the lighter hadrons. The decays sorted out in this way for the various hadrons considered are reported in Table 1. In particular, for the $\Xi(1530)$ and $\Omega(1672)$, the kinds of decay selected have not yet been seen [1], although they are not, in principle, forbidden.

Let us now introduce the power series

\begin{equation}
(1) \quad m = m_0 \{1 + (1/2)\beta + (3/8)\beta^2 + (5/16)\beta^3 + \ldots\}
\end{equation}

which for $0 \leq \beta < 1$ converges to

\begin{equation}
(2) \quad m = m_0 / \sqrt{1-\beta}.
\end{equation}

The value of $m_0$ will be the mass of the pion or the nucleon.
according to whether the decaying hadron considered is a meson or a baryon, respectively.

We can, finally, enunciate our empirical rule. If, for each of the chosen decays, the value of $\beta$ in eq. (1) is such that the sum of the minimum number $N$ of the first terms of the series, with $N \geq |S| + 1$ (or $\geq |C| + 1$) when the decaying hadron has strangeness $S$ (or charm $C$), equals the sum of the masses of the states produced in the decay, then, for that very value of $\beta$, the series will converge to the mass value $m$ of the decaying state (see Table 1).

We may take into consideration also the decays in which hadrons and leptons are produced. In this case, both hadrons and leptons will contribute to form the number $J$ defined in our selecting criterion; and we will select the decay with $J$ minimum under the condition that $J+1$ hadrons and leptons, in all, are produced. An additional decay, only for meson $K$, namely $K^- \rightarrow \pi^0 \mu^- \bar{\nu}$ (or $K^+ \rightarrow \pi^0 \mu^+ \nu$), satisfies those conditions (see Table 1). The above empirical rule applies also to this decay.

Using as external input to our empirical rule the mass values of the pion ($\pi$) and nucleon ($N$), we obtain the mass values of the $K^\pm$, $\Lambda$, $\Xi^0$, $\Xi^-$, $\Xi^0(1385)$, $\Omega(1672)$, and, $\Lambda_c^+$. For the last five hadrons, mass values obtained as output in preceding applications of the rule are required to be used as input. Additionally, using also the experimental mass of the $\Delta(1232)$, we get the mass value of the $\Xi(1530)$. In particular, the mass value of the $K^\pm$ can be obtained independently also from the masses of the pion and muon.

To obtain the values of $m$ reported in Table 1, we have used as $m_0$ in eq. (1) the mass of the $\pi^0$ (=134.96 MeV) for the meson decays, and the mass of the proton $p$ (=938.28 MeV), for the

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baryon decays (with the only exception of the decay $\Lambda \rightarrow n\pi^0$, where $m_0$ was the mass of the neutron $n$). For the $\Lambda$, $m = 1116$ MeV is the average over the two values of $m$ obtained from the two variants (both selected by the above criterion) of the decay considered, i.e. $\Lambda \rightarrow p\pi^-$ and $\Lambda \rightarrow n\pi^0$, with $m$ equal to 1119 MeV and 1113 MeV, respectively. For the $\Xi$ and $\Xi(1530)$, the values of $m$ are average values calculated in the same way. The decay $K^0 \rightarrow \pi^0\pi^0$ is not reported because it does not meet the requirements of the selecting criterion. Accordingly, for the decay $\Lambda^+_c \rightarrow N\pi\pi\pi$, only the variant with $K^-$ has been used to calculate the mass value of $\Lambda^+_c$ (as mass value for $K^-$, to be used as input, we have taken the average value of $m$ from the two decays $K^- \rightarrow \pi^0\pi^-$ and $K^- \rightarrow \pi^0\mu^-\nu$, i.e. 495 MeV). For those hadrons for which $m$ is calculated using preceding outputs of the rule, we have also reported, within brackets, the value of $m$ directly calculated from the real experimental masses of the decay products. For all the strange and charmed hadrons considered, the mass values obtained by means of our empirical rule are in good agreement with the experimental masses.

Incidentally, I wish to report that eq.(2) can be used, in a different context, to point out other extensive regularities concerning the mass values of hadrons. We can, in fact, see that the relative difference of masses squared, $\delta = (m^2 - m_0^2)/m^2$, inside the baryon multiplets with $J^P = 1/2^+, 3/2^+$, and the meson multiplets with $J^{PC}$ in the natural series, i.e. $1^{--}$, $2^{++}$, $3^{--}$, assumes values which follow regularities expressed in terms of a common constant. For each of these multiplets, let us indicate by
the hadron with strangeness $S = 0$ and isotopic spin $I \neq 0$
(having no strange constituents); by $\mathcal{G}_1$ the $|S| = 1$ hadron of
lowest mass (having one strange constituent); and by $\mathcal{G}_2$ either
the baryon with $S = -2$ (containing a pair $ss$), or, for the
mesons, the mostly octet isosinglet with $S = 0$ and $I = 0$ (which
is, in all these cases, composed almost exclusively by a pair $ss$
[2, 3, 4]). If $m_0$ in eq.(2) is now the mass of hadron $\mathcal{G}_0$ in one
of the above multiplets, and $m$ is the mass either of $\mathcal{G}_1$ or $\mathcal{G}_2$ in
the same multiplet, then the corresponding values of $\beta$ fit the
following simple relationship, with a very good accuracy,

$\beta(\mathcal{G}_2)/\beta(\mathcal{G}_1) \approx \varepsilon$

where the constant $\varepsilon = 1.671$ is independent of the multiplet
considered.

4. Concluding Comments

We have defined an empirical rule by which, for certain
specific decays, the mass value of the decaying hadron can be
obtained from the masses of the produced states. By applying
this rule iteratively, the lowest part of the strange and charmed
hadron mass spectrum can be reproduced using as initial input
only the values of mass of the pion (and the muon) and the
nucleon.

We have used above a parametrization of the hadron masses
which is based on a suitably defined relative difference of
masses squared (our parameter $\beta$). Only when this parametrization
is seen in the context of SU(3) multiplets, it produces
coincidences, i.e. the additional empirical relationship
expressed by eq.(3), which reflect well understood regularities corresponding to the strange quark content of the hadrons concerned. On the contrary, the coincidences produced in the context of our empirical rule, which relates the mass of a decaying hadron to the masses of the decay products, appear widely unforeseen.
References

Caption to Table

Table 1. Selected decays for which the value of mass of the decaying hadron can be predicted from the masses of the produced states. The symbol # denotes a possible decay, even if not yet seen. The experimental mass values are taken from Ref.[1].
| Decay                  | J | |S|+1 | N | m (MeV) | m_{exp}(MeV) |
|-----------------------|---|---|-----|---|---------|--------------|
| K^+ → π^0 π^±         | 1 | 2 | 4  | 499| -       | 494          |
| K^− → π^0 μ^− γ       | 2 | 2 | 3  | 491| -       | 494          |
| Λ → Nπ                | 1 | 2 | 2  | 1116| -       | 1116         |
| Σ^0 → Λγ              | 0 | 2 | 2  | 1191(1190)| 1192      |
| Σ → Λπ                | 1 | 3 | 3  | 1316(1314)| 1315-1321 |
| Σ(1385)^0 → Σ^0γ      | 0 | 2 | 2  | 1381(1386)| 1384      |
| Σ(1530) # Λ(1232)π    | 1 | 3 | 3  | 1533| -       | 1532-1535 |
| Ω(1672) # Σ(1385)^0π^-| 1 | 4 | 4  | 1675(1680)| 1672      |
| |                      |   |   |     |     |         |              |
| Λ_c^+ → pK^- π^-      | 2 | 2 | 3  | 2294(2295)| 2281      |

TABLE 1