Offloading Cellular Traffic with Opportunistic Networks: A Feasibility Study

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Abstract—The widespread diffusion of powerful mobile devices with diverse networking and multimedia capabilities, and the associated blossoming of content-centric multimedia services is contributing to the exponential increase of data traffic in cellular networks. Mobile data offloading is a promising technique to cope with these problems, which allows to deliver data originally targeted for cellular networks to complementary networking technologies. Among the various forms of mobile data offloading in this study we focus on offloading through opportunistic networks. Differently from previous studies in this field we evaluate the efficiency of opportunistic offloading schemes by using a real cellular traffic dataset collected in a large metropolitan area over a period of one month. We focus our analysis on video requests for popular video providers, and we evaluate the potential benefits of using an opportunistic data dissemination scheme to request these videos from local users instead of using the cellular network. As a benchmark, we compare the performance of such system with a simple caching mechanism. We show that a simple opportunistic offloading scheme can improve the performance of the caching system even if only 10% of the users participate in the opportunistic dissemination. This means that operators could offload their network efficiently without needing to deploy additional caching infrastructure.

I. INTRODUCTION

Over the last few years we have witnessed an exponential growth of data traffic in cellular networks due to three major trends: i) the proliferation of powerful personal mobile devices, such as smartphones and tablets, with multiple heterogeneous wireless interfaces, ii) the fierce competition among cellular network operators that fostered more affordable data plans for their subscribers, and iii) the diffusion of content-centric multimedia services, such as Netflix, Youtube or Spotify, among mobile users. Recent estimations reveal that this traffic growth is not expected to slow down in the near future [1]. Indeed, mobile data traffic is expected to increase 18-fold from 2011 until 2018, resulting in the so-called data tsunami, and more than 2/3 of this traffic will be related to video consumption. Hence, cellular operators strive to deal with this surge of mobile data traffic by expanding their networks and deploying 4G/LTE technology. Despite this, in the future, cellular network capacity may still lag behind the exponential growth of data traffic, and this may result in problems similar to those arose by recent collapses of 3G networks [2]. Furthermore, as mobile operators roll out more LTE services to meet the growing demand for capacity, with a multitude of devices constantly attempting to connect to remote cloud services the Internet, mobile backhaul systems may also become overloaded. When the backhaul becomes congested, subscribers experience delays or outages in their service. Therefore, it is of paramount importance for the cellular network providers to find new solutions beyond technology upgrades and infrastructure expansion to cope with the effects of the mobile data deluge on both the mobile access and the backhaul. This problem is so crucial that 3GPP standards are for the first time planning to incorporate (among others) capacity improvement mechanisms based on direct Device To Device communications, such as LTE-D2D [3] or PROSe [4]. For this reason, in this paper we focus on data offloading.

Data offloading exploits diversity of wireless access technologies to switch traffic from one access technology to another when deemed beneficial [5]. The idea is to leverage unused bandwidth of complementary network technologies for delivering data originally intended for the cellular network. The main complementary network technology currently used for mobile data offloading is WiFi, and some cellular operators already attempt to offload traffic from the cellular network onto WiFi access points when available. However, different and more sophisticated offloading strategies are also under investigation beyond the simple integration of multiple wireless broadband infrastructures. In particular, in this paper we consider opportunistic offloading strategies that exploit direct communications between mobile devices to relieve congestion in the cellular network [5]. Specifically, in offloading schemes based on opportunistic networks content requests are not initially sent to a cellular base station but they epidemically spread to other user devices by exploiting contact opportunities between nodes that become available over time due to their mobility. If the request reaches one of the other devices that already has the content in its local cache, that device distributes the content to the requestor using the opportunistic network. On the other hand, if the requested content is not already present in the opportunistic network the requestor utilises the cellular network to access the original content. Thus, opportunistic data offloading has the clear advantage of not only reducing congestion on the cellular backhaul but also it significantly reduces the traffic load on the access part of the cellular network. Furthermore, opportunistic data offloading...
does not require additional hardware or functionalities in the network infrastructure since each user device contributes part of its storage as a local cache for temporarily storing copies of content items that are spread in the opportunistic network.

To the best of our knowledge, opportunistic offloading has only been evaluated through analysis or simulations but it has not been tested on real cellular network traffic. In this paper we contribute to fill in this gap by using a real world cellular traffic dataset collected in a large metropolitan area over a period of one month, and we focus our analysis on video requests for popular video providers. The main goal of our study is to understand the maximum achievable traffic reduction in our dataset through an opportunistic offloading scheme. Therefore, we first study the maximum cacheability for those contents that are popular (i.e. contents requested multiple times in a give time period) and we use it as reference point to evaluate the offloading approach. Then we show how performs an infrastructure-based caching mechanism and, finally, we show the performance of an opportunistic offloading system based on opportunistic networks. Specifically, we compare the gain in traffic reduction brought about by opportunistic offloading w.r.t. a simple caching system. The considered caching system leverages on global knowledge about the network to supervise and coordinate the entire content delivery. However, the deployment and the maintenance of such infrastructure can be expensive for the network operator, and the access network at the edge of the cellular network is still used to access the cache server. With respect to caching, an offloading system has the advantage of being entirely distributed between the mobile devices: content is stored in mobile devices, which all contribute to the offloading system, and hence discharge the operator from deployment and maintenance costs together with the redaction of traffic coming from the access to the cellular network. Surprisingly, our results show that opportunistic data offloading can be up to 20% more efficient than caching even if only 10% of the users participate in the opportunistic dissemination. As we discuss in the paper , this is achieved thanks to the users mobility which helps the diffusion of contents in the city.

The remainder of this paper is organised as follows. First, we review a set of related works exploring different offloading techniques, before presenting our dataset in Section III. We describe the opportunistic offloading methodology in Section IV, we show the results of our analysis in Section V, and discuss our findings in Section VI.

II. RELATED WORKS

It is out of the scope of this section to provide an exhaustive review of the various approaches that have been proposed of far to support the mobile data growth in cellular systems. Thus, we focus only the studies that are most related to our work.

Early studies explored caching of web resources by defining static content as cacheable, and dynamic content as not cacheable. Cao and Irani [6] conducted a trace-driven study of web cacheability and showed that up to 50% of the content can be cached. But the internet in general and the web in particular have since evolved and the content available is very different from the content studied in these early works. For instance, user generated content (UGC) websites are now a large part of user activities, with websites offering a large variety of contents generated by users, such as question-answer databases, digital video, blogging, podcasting, forums, reviews, social networking, social media, or mobile phone photography. The amount of static content (not dynamically generated) represents a much higher proportion of the traffic nowadays, and it is assumed that a large proportion of the content is potentially cacheable. Therefore, the cacheability is now simply defined by recent studies as the fraction of requests that can be answered by a cache. In particular, traffic dataset have been analysed to study cacheability of videos on fixed network [7]–[9], and more recently on cellular networks. For example, Guillemin et al. [10] study the cacheability of YouTube videos with a single cache server for each of the three cities covered by their traffic trace dataset. Ramanan et al. [11] also studied the cacheability of YouTube videos in a live cellular network over 24 hours. Ben Abdesslem and Lindgren [12] recently evaluated the performance of caching YouTube videos on a nationwide cellular network, showing that 21% of the requests could be delivered by a cache server at each cell.

Mobile data offloading can take several forms. There are several studies that have investigated the offloading efficiency when mobile devices can offload their data traffic from cellular networks onto WiFi networks whenever WiFi networks are available. For instance, a quantitative study is presented in [13] by using traces of WiFi connectivity in large metropolitan areas. The authors discovered that if data transfers can be delayed with some deadline until users enter a WiFi zone, substantial gains can be achieved only when the deadline is fairly larger than tens of minutes. Kim et al. [14] develop an analytical framework to diverse offloading scenarios, and show that in a metropolitan city, 80% of the traffic can be offloaded in certain conditions. However, the efficiency of this kind of offloading schemes rapidly degrades if the WiFi infrastructure is not sufficiently dense. Thus, recent initiatives are promoting opportunistic local communications through shared WLAN access points that can allow information exchange even when the Internet infrastructure is not available [15], [16]. Most related to this work are the several studies that investigate offloading through opportunistic networks. Han et al. [17] (then subsequently extended in [18]) were the first to exploit opportunistic communication to alleviate data traffic in the cellular network. In their seminal paper they exploit information about the contact graph of mobile nodes in order to select the initial set of k mobile nodes in charge to trigger the opportunistic dissemination of the requested content. In [19] the authors suggest exploit to social aspects of user mobility to select the most socially important nodes to be used for spreading contents in the network. In [20], it is proposed Push&Track (PT), an offloading hybrid solution in which a central dissemination controller selects the number of content replicas to inject in the opportunistic network trying to follow predefined performance targets. Rebecchi et al. [5] proposed Droid (Derivative Re-injection to Offload Data), an autonomic version of PT which adaptively estimates the actual dissemination trend in the opportunistic network, evaluates the evolution of dissemination level for future time steps and, if needed, injects new content replicas in order to reach the total diffusion within a time limit. Finally, in [21] is proposed an offloading solution that shares the same architectural design
of Droid and PT but uses an Actor-Critic learning system to decide the time instant and the number of content replicas to be injected in the network. Its purpose is to maximise the opportunistic diffusion of contents, while minimising the overall number of content injections operated by the central controller. A more complete survey about offloading solution based on opportunistic networks can be found in [22]. To the best of our knowledge, there are not previous papers that have used real traces of cellular traffic to assess the performance of an opportunistic data offloading scheme.

III. DATASET

A. Dataset Description

The analysis presented in this paper is based on a large-scale dataset of cellular network traffic traces from a major European operator\(^1\). The traces were collected on a national scale over a period of 41 days, from 16th December 2011 at 17:00 to 25th January 2012 at 18:00. The dataset contains the URLs of several tens of billions HTTP requests together with timestamps, content sizes, and anonymised identifiers of the users and the cells.

Out of the 961 hours between the first and last request collected, 9 hours are missing from the dataset: two hours on 21st December, three hours on 2nd January, two hours on 22nd January, and one hour on 23rd and 25th January respectively. Although those missing hours are noticeable, for instance, when looking at the amount of requests on the days with missing hours, we believe that they are not changing any of our conclusions.

In our analysis we focus on video requests sent to the 10 most popular video providers in the dataset. Since we are interested in studying data offloading, we limit our analysis to the activity within a large city with around 1,000 cells deployed over 20 districts. This is typically a dense environment, both in terms of number of users and rates of data traffic demand, and is therefore very suitable for offloading techniques. The dataset contains around one million HTTP requests targeting 524,787 unique videos, and generated by 398,329 unique users. According to the video content specified in HTTP requests, we identified two main classes of content requests: regular and adult. Precisely, in our dataset, the 88\% of the total number of requests is for regular content while the remaining 12\% is for adult content. It is worth noting that this is just one possible traffic differentiation and we use it to show that different traffic classes can be more or less suitable for offloading.

The content requested in the dataset present very different levels of popularity. Popularity of a content item is defined here as the number of times it has been requested. The performance of offloading schemes is very sensitive to content popularity: the more a content is requested in a given time window, the more cellular traffic can be saved through an offloading mechanism. We analyse hereafter the popularity of content (both regular and adult) requested in the dataset, and we derive the corresponding statistical distribution. As can be seen in Figure 1a-1b, the content popularity distribution shows a Zipf-like behaviour. In order to statistically confirm this assumption, we fitted the empirical distribution of the number of content requests with a Zipf distribution with parameter \(\rho\).

\(\text{TABLE I. Pearson }\chi^2\text{ goodness of fit test with a Zipf distribution (95\% of confidence level).}\)

<table>
<thead>
<tr>
<th>Content type</th>
<th>Pearson (\chi^2) Test Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Statistic</td>
</tr>
<tr>
<td>Regular</td>
<td>0.000085</td>
</tr>
<tr>
<td>Adult</td>
<td>0.002666</td>
</tr>
</tbody>
</table>

The maximum likelihood parameter estimates are \(\rho_r = 1.47\) and \(\rho_a = 1.39\), for the regular and adult traffic, respectively. Moreover, we applied a typical statistical test, the Pearson \(\chi^2\) hypothesis test [23], in order to confirm, with a confidence level of 95\% that the fitted distribution is compatible with the dataset. In both cases the null-hypothesis has not been rejected, thus we can assume with a reasonable confidence level that the content popularity in this dataset follows a Zipf distribution, as reported in Table I. It is noticeable, however, that the tail of the distribution for the regular contents is heavier than the one for adult contents, i.e. regular contents are more requested that adult ones. As we show in Section V, this has a remarkable impact on the offloading performance. We point out that although popularity of contents is commonly assumed Zipf-like distributed, performing analysis on real traces is still important.

\(^1\)Due to a confidential disclosure agreement we can not provide information that could permit to identify the cellular operator.
B. Maximum achievable efficiency and performance metrics

As pointed out in previous section, a large number of videos are requested only once. In this scenario, when a content is requested only once, no offloading would be useful, as the content will not be requested again. Therefore, in order to fairly evaluate the infrastructure-based caching and the opportunistic offloading mechanisms, we define the cacheability metric as the maximum efficiency that can be achieved in an ideal case by any offloading. Precisely, it expresses the offloading level achievable if all traffic can be offloaded, except from the first request, and it is computed according to the following equation:

$$\text{cacheability} = \frac{\sum_{c \in C} (k_c - 1)}{\sum_{c \in C} k_i},$$

where $k_c$ is the number of times the video $c$ has been requested and $C = \{c : k_c > 1\}$ is the set of videos that have been requested more than once. Note that the above cacheability metric has been already used by Ager et al. in [7] and other related works [11], [24]. Note that in this study the cacheability index is always computed by considering requests generated within the same city district, i.e. requests generated in different districts do not contribute to the value of the same index.

From the perspective of the cellular network operator, the gain from offloading mechanisms is expressed as the proportion of total offloaded traffic over the total traffic. But from the end users’ perspective, we can imagine an offloading service for popular content only, to guarantee the local availability of popular content and, since there is less content to store, to reduce the local cache sizes. For this reason, and to understand the impact of content popularity on the opportunistic offloading, we also measured the performance of the offloading systems taking when offloading is applied only to the content items that are requested more frequently. More precisely, in the following sections we show results that are obtained when only a subset of most popular videos that corresponds to the 10%, 30% and 50% of the total number of requests present in the dataset are considered for caching and offloading. For the sake of brevity, we refer to such subset as traffic cuts. Finally, we analysed regular and adult traffic separately, as the request patterns and popularity characteristics might be different. As we will show in the next section, the offloading system we consider allows content providers to measure the popularity of contents, therefore, the identification of popular contents is viable in practice with small additional efforts.

IV. OPPORTUNISTIC OFFLOADING METHODOLOGY

In principle, the feasibility of an offloading mechanism based on opportunistic networks depends on several factors: i) users must rely on ad hoc enabling technologies, like WiFi Direct or Bluetooth to enable direct communication between them, ii) users’s mobility patterns should ensure that a sufficient number of contact opportunities between different nodes occurs, and iii) requested contents must be popular, i.e., they are requested by many users within the same geographical area. In Section III-A we have already verified the third condition as the content popularity in our trace follows the typical Zipf-like behaviour.

In this study we consider a simplified opportunistic offloading mechanism. Specifically, mobility information is not available from our dataset, therefore we assume a simple dissemination process that models the communication between users through which they are able to both send requests and disseminate contents into the opportunistic network. Precisely, we assume a dissemination process that reaches a certain fraction of nodes inside the district within a short amount of time (w.r.t. the inter-request times of content requests). According to [25], such model, though simple, is reasonable for a dense scenario with high contact rates as a city district can be. In our system, the fraction of nodes that can be reached by the dissemination process is identified by the parameter $\mu \in [0, 1]$. Precisely, the parameter $\mu$ is used to model the unreliability that it is intrinsic to the opportunistic dissemination process. However, it can be simulated stochastically by drawing at random the fraction of nodes that, at any given point in time, are able to exchange requests and content through multi-hop communications. In this paper, we assume that the request or the diffusion of a content is implemented through a simple epidemic diffusion algorithm [26]. To this end, users’ mobile devices contribute with a limited cache to the dissemination process because they locally store contents that they receive from other encountered users, even if they have not requested that content. However, to limit the cache size we also assume that contents have a limited time validity, after which they are dropped from the node’s cache. From now on a not yet expired content will be referred as valid content.

In this paper we adopt an opportunistic data offloading scheme similar, in principle, to the Push & Track solution [20]. Specifically, we assume the existence of a central dissemination controller that monitors and drives the offloading process. The central dissemination controller maintains the list of valid contents spread in the opportunistic network at a given time, and their associated expiration timeout. Furthermore, we assume that the central controller can communicate with the mobile devices through lightweight control messages, which are used to check and update, by nodes, the expiration timeout of stored contents.

The opportunistic offloading mechanism we consider works as follows. Every time a new content is requested by the a user $A$, it disseminates the request to a fraction $\mu$ of the other users within the same district. If one of the users receiving the content request has a valid copy of the content in its cache (i.e. not yet expired), it sends the content to the originator of the request. Moreover, the content is epidemically disseminated to all other nodes that can be reached by the node $A$. Otherwise, if none of the contacted nodes has the requested content in their cache, node $A$ downloads the content from the cellular network. Furthermore, node $A$ immediately disseminates the received content to a random fraction $\mu$ of nodes belonging to the same district. In any case, every time a node receives a content, the central controller is notified and it reset the expiration timer associated to the content. It is worth noting that this kind of central controller only keeps track of the validity of a content and it does not store the content itself. Therefore this central controller is much more simple and less expensive to implement w.r.t. a typical infrastructure-based caching system.
V. PERFORMANCE EVALUATION

Before analysing and evaluating how opportunistic data offloading performs, we first present and evaluate the efficiency of an infrastructure-based caching system. Then, we use the results of this performance analysis as a reference benchmark for the efficiency of the opportunistic data offloading scheme.

A. Caching methodology

From a topological point of view, we consider a scenario in which a certain number of cache servers are installed by the operator in the cellular network, with each server covering users in a limited geographical area. These cache servers store the content requested by the users in that area. Each content is stored for a limited time window, preliminary set by the operator, after which they are dropped from the cache. Every time a content is requested, its expiration time is extended for an additional period of time, equal to the time window. Moreover, in the considered scenario we assume that cache servers are equipped with a storage large enough to contain any content until they expire.

This simple caching mechanism works as follows. Every time a user requests a content from the cellular network, the cache server intercepts the request and looks for the content in its local storage. If the content is locally stored then it is directly downloaded from the cache server by the user’s mobile phone; we refer to this event as a hit. Conversely, if the cache server does not have a local copy of the requested content, then it is considered a miss because the server has to download the content from the Internet, store it, and send it to the user.

B. Caching results

In this section we present the performance that can be expected from this caching mechanism. The analyses we present refer to the performance of a caching infrastructure operating in a geographical area having the size of a city district. In our dataset there are 20 districts we assume being served by a cache server, each. As in most previous studies on caching found in the literature, we use the hit rate as performance metric. Furthermore, we consider the hit rate both in absolute and relative terms. Specifically, the absolute hit rate for a cache server refers to the number of hits divided by the total number of requests received by this server, while the relative hit rate of a traffic cut is the number of hits divided by the number of requests for videos in that traffic cut. Both hit rates are given for different time widows, and for both regular and adult content. We point out that we present results in an aggregated way among all districts. Interestingly, although the regular and adult traffic have a very different impact on the entire generated traffic – 88% of the requests are targeting regular content – the performance of the caching system is similar for both types of content. In fact, in Figures 2a and 2b the hit rate curves for the different traffic cuts show very similar trends. This suggests that, although regular and adult traffic are significantly
different in content types, from an operator point of view it is not necessary to treat them separately by applying specific caching strategies. From the shown results we can also observe that the larger the traffic cut, the higher the relative hit rates. This behaviour can be explained by noting that small traffic cuts include very popular videos that have short Inter-Request-Times. A cached content that is frequently requested remains in cache for longer than less popular content, since their expiration time keeps being extended. For the same reason, hit rates increase as we increase the time window because cached content is discarded less frequently. When we consider absolute hit rates, the order of the curves is inverted, as shown in Figures 2c and 2c. Indeed, the larger the traffic cut, the higher the hit rates. This is an expected result since the smaller the traffic cut, the smaller the fraction of total requests that are served by the cache server. Results in Figures 2c and 2c are also interesting from an operator point of view because they show how much traffic can be saved through a district-wide caching mechanism. Moreover, it is possible to see the actual impact of the different time windows on the caching performance. Storing contents for longer times significantly improves the performance of the caching system, although a larger time window also usually corresponds to a more expensive caching system.

C. Offloading results

For the sake of figure clarity in the following graphs we report results for two traffic cuts, namely 10% and 100%, and for \( \mu = 0.1 \). Note that \( \mu \) provides an idea of the overhead associated to the opportunistic mechanism, as only a fraction \( \mu \) of nodes are involved for each content request. In order to have a fair comparison with the caching system we only consider the district-wide caching scheme as opportunistic dissemination over the entire city may be unlikely. As shown in Fig. 3c-3d, similar trends are obtained with the opportunistic offloading as with caching for the absolute hit rates. However, opportunistic offloading can be up to 200% more efficient than caching at the district level and for regular traffic, even if only 10% of the users participate in the opportunistic dissemination. This can be explained by noting that users’ mobility can allow the circulation of less popular contents between different city districts. Thus, a content can be available in the local cache of one of the nodes in a community if it was not previously requested by other nodes in that same community. This interesting result from an operator point of view because it demonstrates that it is not necessary to deploy expensive caching infrastructures to support data growth, but relying on the (limited) storage resources of its subscribers may be feasible. Note that in the adult case, opportunistic offloading
and caching perform the same because the the distribution of the popularity of adult videos is much more flat than of regular videos.

Relative hit rates are shown in Fig. 3a and 3b for regular and adult traffic, respectively. As for caching, we can observe that the smaller the traffic cut, the higher the relative hit rates. However, in this case we can also notice that there is a noticeable difference between the regular traffic and the adult traffic. Specifically, for adult traffic caching and opportunistic offloading perform almost identically also as far as relative hit rates are concerned. On the other hand, for regular traffic opportunistic offloading largely outperforms caching solution. Again, this can be explained by recalling that the difference between the number requests for the most popular video and for the least popular video is one order of magnitude less than in the case of regular traffic. In our offloading scheme, the central controller reset the operation timeouts of a cached content whenever there is a request for that content. Thus, the most popular contents tend to be more persistent in the network and this facilitates the opportunistic dissemination.

VI. CONCLUSIONS REMARKS

Although many studies exist in literature to evaluate to which extent a cellular network operator can use offloading techniques to deal with the exponential traffic growth problem, most of them rely on simplifying assumptions or are targeting ideal deployments. Our study adopts a radically different approach, since we have performed a trace-based feasibility analysis of two different, and somehow complementary, solutions – caching and opportunistic data offloading – using the real pattern requests of videos for popular video providers in a large metropolitan area. The two most important results of our analysis can be summarised as follows: i) given the content popularity and the request frequency it is enough to cache content for one hour to obtain satisfactory offloading efficiencies, and ii) opportunistic offloading, when applied to a restricted area, is as least as efficient as an infrastructure-based caching system.

We believe that this study opens interesting directions for future research. First, we intend to investigate more sophisticated data dissemination strategies beyond simple epidemic-based diffusion algorithms, which take into account the spatial and temporal distributions of content requests. Second, we want to extend our performance analysis to include more accurate mobility models and more realistic constraints on the opportunistic communications (e.g., limited bandwidth, limited contact duration, etc.)

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