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ABSTRACT

Vehicular Ad Hoc Networks (VANETs) are attracting the attention of researchers, industry, and governments for their potential of significantly increasing the safety level on the road. In order to understand whether VANETs can actually realize this goal, in this paper we analyze the dynamics of multi-hop emergency message dissemination in VANETs. Under a probabilistic wireless channel model that accounts for interference, we derive lower bounds on the probability that a car at distance d from the source of the emergency message correctly receives the message within time t. Besides d and t, this probability depends also on 1-hop channel reliability, which we model as a probability value p, and on the message dissemination strategy. Our bounds are derived for an idealized dissemination strategy which ignores interference, and for two provably near-optimal dissemination strategies under protocol interference. The bounds derived in the first part of the paper are used to carefully analyze the tradeoff between the safety level on the road (modeled by parameters d and t), and the value of 1-hop message reliability p. The analysis of this tradeoff discloses several interesting insights that can be very useful in the design of practical emergency message dissemination strategies.

1. INTRODUCTION

Vehicular Ad Hoc Networks (VANETs) have recently attracted the attention of researchers, automotive industry, and governments, for their potential of improving driver's awareness of the surrounding environment through infrastructure-less, vehicle-to-vehicle (V2V) wireless communications. The major driving factor for the investigation and deployment of this new technology is safety: by collecting accurate and up-to-date information about the status of the surrounding environment, a driver assistance system can quickly detect potentially dangerous situations, and notify the driver about the approaching peril. Since a relatively small reduction in the driver response time (in the order of a fraction of a second) can result in avoiding an accident, driver assistance systems based on V2V communications have the potential of significantly improving the safety level on the road.

While improving the safety level on the road is unanimously considered the major driving factor for the deployment of VANETs (see, e.g., [6]), the challenges related to implementing safety applications (e.g., collision avoidance system, hazard warning, etc.) are the most difficult to solve in the area of vehicular networking. In fact, safety applications must rely on *very accurate* and *up to date* information about the surrounding environment, which, in turn, requires the use of accurate positioning systems and smart communication protocols for exchanging information. These smart communication protocols should guarantee *fast* and *reliable* delivery of information to *all* vehicles in a neighborhood, in a network environment in which the communication medium is shared, highly unreliable, and with limited bandwidth.

It is clear from the above description that a fundamental building block of any safety application for VANETs is a reliable and time constrained 1-hop broadcast primitive, which is not currently included in the emerging DSRC (Dedicated Short Range Communications) standard [5] (a variation of IEEE 802.11a). A recent study has shown that the DSRC standard provides adequate performance for what concerns delay, but it is defective in terms of reliability [17]. This explains the many recent proposals for modifications of the MAC layer aimed at improving either reliability, or reducing delay, or both, of 1-hop broadcast [2, 14, 15].

Another important building block of safety applications is prioritization of messages: safety applications will coexist with non-safety applications such as traffic information systems, commercial services, and with less critical safety-related messaging (beaconing). Although a separate control channel is used in DSRC for safety-related applications, these applications still have to contend for the channel in case cars are equipped with a single radio (as it will typically be the case). As a consequence of this, it is important to ensure that, when a safety application needs to access the channel, it gets a higher priority than non-safety related or less critical safety-related messages. Prioritization of safety-related messages is investigated in [14, 11, 12, 13].

In this paper, we are concerned with one specific safety application: hazard warning dissemination[1]. In this type of application, a certain vehicle issues a hazard warning message (also called *emergency message* in the following) in case a dangerous situation is detected (e.g., obstacle on the road, airbag explosion, malfunctioning of the braking system, and so on). This emergency message should be propagated backward on the road as quickly and reliably as possible, in order to enable the drivers of approaching vehicles to undertake adequate countermeasures. Fast backward propagation is needed because the information contained in the emergency message has a very limited lifetime (i.e., it is useful only if received within a short time from the hazard warning inception); reliable propagation is needed because a single vehicle

[1] Indeed, the study reported in this paper is relevant to all safety applications which require fast and reliable (multi-hop) dissemination of emergency messages.

missing the warning can become extremely dangerous for the other vehicles. Observe that, since the typical transmit range for reliable communication in VANETs is in the order of a few hundred meters, multi-hop emergency message dissemination is needed in this application scenario to increase the area of hazard warning delivery.

The main goal of this paper is to investigate the fundamental tradeoff between "safety-level" and "emergency message resource wastage" encountered in hazard warning applications. A common feature of several techniques proposed in the literature to improve reliability of 1-hop emergency message broadcast is allowing the tuning of the amount of bandwidth reserved for emergency message dissemination (e.g., by setting the number of MAC layer repetitions [14], or the transmit power level [12, 13]): by devoting more resources to safety-related message dissemination, reliability of 1-hop broadcast can be increased. On the other hand, this comes at the expense of less available bandwidth for non safety-related applications, which is reduced accordingly. To the best of our knowledge, the question of how to adequately address this tradeoff has been evaded in current literature.

Note that a major difficulty in the investigation of the above described tradeoff is analyzing the relation between 1-hop reliability (which is relatively easy to estimate – see [11, 13]) and multi-hop, time-constrained reliability. In fact, multi-hop, time-constrained reliability is influenced by several factors (including, of course, 1-hop reliability), such as network topology, channel characteristics, interference, message dissemination strategy, and so on.

As a first step towards an in-depth investigation of this tradeoff, in this paper we fix the network topology, channel characteristics and interference model, and we study the dependence between 1-hop and multi-hop, time-constrained reliability under various message dissemination strategies. Our study uses a probabilistic approach, and includes time constraints on hazard warning reception. More specifically, we estimate the probability $P(\bar{d}, \bar{t}, p)$ that a vehicle located at distance $\bar{d}$ from the hazard warning initiator correctly receives the warning within time $\bar{t}$, where p is a parameter modeling 1-hop reliability. Based on this estimation, we gain several insights on the "safety-level"/"emergency message resource wastage" tradeoff. In particular, we find that the relative advantage of having an increased 1-hop reliability value (which might come at the expense of considerable resource wastage) tends to decrease as the distance from the emergency message initiator increases. Furthermore and more interestingly, the efficacy of increased 1-hop reliability values displays a clear dependence on the traffic conditions: under heavy traffic conditions, relatively low values of 1-hop channel reliability can be used without considerably impacting multi-hop reliability; on the contrary, under light traffic conditions high values of 1-hop channel reliability turn out to be very useful to improve multi-hop reliability. We also find that the emergency message dissemination strategy has a major impact on $P(\bar{d}, \bar{t}, p)$. Finally, we have verified through simulations that our derived bounds on $P(\bar{d}, \bar{t}, p)$ are qualitatively very accurate.

To the best of our knowledge, the one reported in this paper is the first attempt to systematically analyze time-constrained, multi-hop reliability in vehicular ad hoc networks. A few other papers in the literature are concerned with multi-hop message propagation in VANETs, but they are either not concerned [3, 8, 10] or only partially concerned [16] with delay in message propagation.

2. NETWORK AND CHANNEL MODEL

In this section, we introduce the network and channel model used in our analysis of multi-hop emergency message propagation in VANETs. Due to the inherent difficulties of this analysis, we use a simplified network and channel model, which, however, capture relevant VANET features.

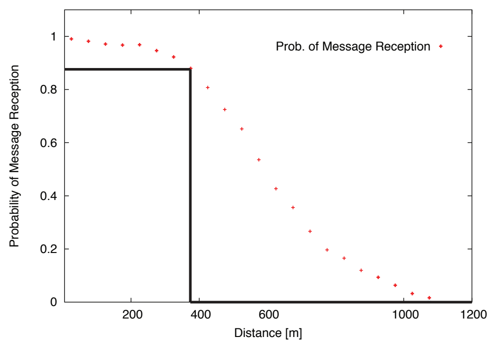

Figure 1: Example of how our channel model can be used to lower bound message reception probability. Actual message reception probabilities are computed assuming a nominal communication range of $1000m$, and Nakagami radio propagation (courtesy of M. Torrent-Moreno).

We consider a vehicular ad hoc network in which nodes (cars) are equally spaced along a line, where d is the separation distance between consecutive cars. When a car transmits a message, all nodes within distance r_T (*transmit range*) from the sender have the same probability p $(0 < p < 1)$ of correctly receiving the message in absence of interference. To ease presentation, in the following we normalize the transmit range and all the other relevant distances with respect to the inter-vehicle distance d (i.e, we assume $d = 1$). Since we are concerned with life critical message dissemination, which is of utmost importance to all cars on the road, we assume that all messages are sent in broadcast mode.

Note that the above channel model, although very simple, captures the most relevant feature of wireless transmission, i.e. uncertainty about correct message reception. A very similar channel model is used in [16] to estimate the performance of a cooperative collision warning protocol. As shown in Figure 1, the inaccuracy of our channel model with respect to more realistic ones goes 'in the right direction', i.e., it underestimates correct message reception probability. How to extend the analysis reported in this paper to more accurate probabilistic channel models is matter of ongoing research.

To model interference between concurrent transmissions[2], we use the protocol model introduced in [7]: for a given node u, a message is correctly received by u with probability p if and only if there exists a transmitter within distance r_T from u, and no other node within distance $r_I \geq r_T$ (r_I is the *interference range*) from u is concurrently transmitting.

To analyze message propagation, we use the concept of token: initially, only the rightmost car in the network (the one ahead of the cloud of cars, which is called the *initiator* in the following) has the token (hazard warning to disseminate); the token is then disseminated in the network through a sequence of (possibly concurrent) 1-hop broadcasts; when a node first receives the emergency message, it gets a copy of the token, and it becomes a potential forwarder of the token. To simplify the analysis, we assume that token propagation proceeds in rounds, and that transmissions are carefully scheduled in each round to speed-up message propagation as much as possible.

We are interested in estimating, for a given node u at a cer-

[2]In this paper, we are not concerned with external interference.

tain distance $\bar{d}$ from the initiator, the probability $P(\bar{d}, \bar{t}, p)$ that u gets the token within time $\bar{t}$. Note that the above probability clearly depends on the distance $\bar{d}$ from the initiator, on the reference time $\bar{t}$, and on the probability p of correctly receiving a 1-hop broadcast message in absence of interference. The goal of our analysis is to gain insights on the interdependence of these three parameters, which represents the fundamental tradeoff between safety level on the road and resources devoted to the emergency message dissemination mechanism: by tuning parameters $\bar{d}$ and $\bar{t}$, we can set our required safety level, which can be expressed as a set of $(distance, time)$ pairs (e.g., cars at distance $200m$ from the dangerous event should receive the message within $0.5sec$, cars at distance $300m$ should receive the message within $0.8sec$, and so on). On the other hand, the value of parameter p is determined by the amount of wireless resources (bandwidth) which are allocated to the emergency message dissemination mechanism: the more resources are allocated for emergency message dissemination, the higher the value of p. Note that most of the techniques proposed in the literature to realize reliable emergency message delivery allow to control the amount of consumed wireless resources, e.g. by changing the transmit power level [12, 13], or by changing the number of MAC-layer repetitions of a message [14]. Since allocating resources to emergency message dissemination necessarily reduces the available resources for other VANET services (traffic report, commercial services, and so on), the network designer is interested in finding the minimum value of p that guarantees the required safety level on the road. To the best of our knowledge, the study reported in this paper is the first attempt to carefully analyze this fundamental tradeoff in vehicular ad hoc networks.

3. THEORETICAL ANALYSIS

In the following, we denote a set of n equally spaced cars on a straight line road as a string S_n of bits, where the least significant bit corresponds to the initiator, and the i-th bit to the i-th car in the cloud. In the following, we assume that cars are numbered from 0 (the initiator) to $n-1$, and are ordered accordingly. Without loss of generality, we assume that the initiator is the rightmost node in the cloud of cars. We also use notation $i, j, \ldots$ to denote both a generic node in the network and its position in the cloud of cars.

Bits encode whether a certain car has received the token (bit value is 1), or not (bit value is 0). In the following, we call a node holding the token a 1-node, and a node which has not yet received the token a 0-node. The value of S_n evolves with time, as the token is propagated throughout the network. We express this dependence with time with notation $S_n(t)$, where t denotes the round index.

We assume that initially only the initiator holds the token, i.e., $S_n(0) = 0\ldots01$. During the first round, the initiator transmits the message, and all nodes within distance r_T (i.e., within r_T positions) from it become a 1-node with probability p. At the generic round k, a certain number of transmitters is selected among the 1-nodes, and some of the 0-nodes (which are determined according to the radio channel and interference model) become a 1-node with probability p.

The goal of our analysis, re-stated in terms of bits and rounds, is to estimate the probability that a certain bit turns into a 1 within a certain number $\bar{t}$ of rounds. It is clear that this probability depends also on the strategy used to select transmitters at each round. In this paper, we are interested in characterizing the behavior of the emergency message propagation process under the best possible conditions. This characterization is useful in understanding which is the best possible achievable safety level for a certain value of the probability p of correct message reception, thus serving as a guideline to address the "safety level"/"emergency message resource wastage" tradeoff mentioned in the previous section.

Studying life critical message propagation under the best possible conditions requires identifying an optimal strategy for selecting the set of transmitters at each round, which is a very difficult problem. To circumvent this problem, in the following we define a simple multi-phase strategy to select transmitters at each round, and we formally prove that this strategy is within a constant factor from the optimal strategy.

3.1 Dissemination strategies

Before presenting the strategy and proving the approximation bound, we need the following definition:

Definition 1 (Internal 0-nodes). *Let i be the i-th car in the cloud, and assume i is a 0-node. Node i is said to be an internal 0-node at time t if and only if there exists a 1-node in $S_n(t)$ in position j, with $j > i$.*

The strategy used to select the transmitters set at each round is reported in Figure 2. We call this strategy Global, because it is a centralized strategy that uses global knowledge: the set of transmitters for round h and $h+1$ (or h, $h+1$, $h+2$) is computed by knowing the complete status of $S_n(t)$ at time $h-1$.

The strategy proceeds in stages, where every stage is composed of a certain number of rounds, depending on the relative value of r_I with respect to r_T. A stage takes as input string $S_n(t)$, and outputs sets $T_1, \ldots, T_i$ of transmitters at round $t+1, \ldots, t+i$. We have $i = 2$ if $r_I = r_T$, and $i = 3$ if $r_T < r_I \leq 2r_T$.

In the first round of the stage, the leftmost 1-node (call it node k) is selected to transmit, and a number of other 1-nodes are selected for concurrent transmission according to a greedy rule designed to avoid interference. At the end of the first round, nodes are marked as *covered* if they are within distance r_T from one of the selected transmitters, and as *uncovered* otherwise. Then, groups G_z of consecutive uncovered nodes of cardinality at most r_T are formed by scanning all nodes starting from the right.

The second round of the stage differs slightly depending on the value of the interference range. If $r_I = r_T$, all groups are scanned from right to left and, for every considered group G_z, a 1-node among the nodes in G_z is scheduled for transmission and included in T_2. If all nodes in G_z are 0-nodes, the closest 1-node j to the left of G_z is scheduled for transmission and included in T_2.

In case $r_T < r_I \leq 2r_T$, groups are alternately marked as $\sharp 2$ or $\sharp 3$ at the beginning of the round, and the above described procedure is applied to the groups marked as $\sharp 2$.

Round 3, which is executed only if $r_T < r_I \leq 2r_T$, is equal to round 2, except that transmitters are selected only for groups marked as $\sharp 3$ in the previous round.

We now prove that Global is within a constant factor from the optimal propagation strategy. Since we are dealing with a stochastic process, we need an appropriate definition of optimal strategy, and of approximation bound for non-optimal strategies.

We start observing that our goal is ensuring *fast* and *reliable* backward propagation of emergency messages in the cloud of cars: with fast we mean that the token should be propagated backward as quickly as possible; with reliable, we mean that backward propagation should not come at the expense of leaving many uncovered nodes between the initiator and the front of the emergency message propagation area.

Given this observation, consider string $S_n(t)$ at the generic round t, and denote by k the leftmost 1-node in $S_n(t)$. It is immediate that an optimal strategy cannot do better than the

```
INPUT: S_n(t)
OUTPUT: set T_h of transmitters for round t + h

Bit(S_n(t), i) returns the i-th bit of S_n(t)
Array Tag[ ] stores nodes' tags

ROUND 1
k = leftmost 1-node in S_n(t)
for i = 1 to k do
   Tag[i] = uncovered
end for
T_1 = {k}
i = k − (r_T + r_I + 1)
repeat
   while Bit(S_n(t), i)=0 do
      i = i − 1
   end while
   T_1 = T_1 ∪ {i}
   i = i − (r_T + r_I + 1)
until i > 0
for each i ∈ T_1 do
   for j = 0 to r_T do
      Tag[i + j] = covered
      Tag[i − j] = covered
   end for
end for
z = 1
G_z = ∅
i = 0
while i < k do
   if Tag[i] = uncovered then
      G_z = {i}
      j=1
      while (Tag[i + j] = uncovered) ∧ (j < r_T) do
         G_z = G_z ∪ {i + j}
         j = j + 1
      end while
      h = h + 1
      G_z = ∅
      i = i + j
   else
      i = i + 1
   end if
end while

ROUND 2 (case r_I = r_T)
T_2 = ∅
for each G_z do
   if exists 1-node in G_z then
      j = leftmost 1-node in G_z
      T_2 = T_2 ∪ {j}
   else
      i = leftmost 0-node in G_z
      find closest 1-node j with j > i
      T_2 = T_2 ∪ {j}
   end if
end for

ROUND 2 (case r_T < r_I ≤ 2r_T)
T_2 = ∅
Alternately mark groups G_z as ♯2 and ♯3
for each G_z do
   if G_z is marked ♯2 then
      if exists 1-node in G_z then
         j = leftmost 1-node in G_z
         T_2 = T_2 ∪ {j}
      else
         i = leftmost 0-node in G_z
         find closest 1-node j with j > i
         T_2 = T_2 ∪ {j}
      end if
   end if
end for

ROUND 3
(Needed only if r_T < r_I ≤ 2r_T)

T_3 = ∅
for each G_z do
   if G_z is marked ♯3 then
      if exists 1-node in G_z then
         j = leftmost 1-node in G_z
         T_3 = T_3 ∪ {j}
      else
         i = leftmost 0-node in G_z
         find closest 1-node j with j > i
         T_3 = T_3 ∪ {j}
      end if
   end if
end for
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Figure 2: The GLOBAL strategy.

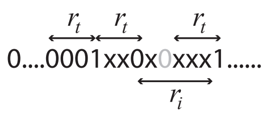

Figure 3: Ensuring both properties i) and ii) of the IDEALIZED strategy might be infeasible due to interference: the leftmost 1 must transmit at each round to ensure property ii); as a consequence, the internal 0-node highlighted in gray has probability 0 (rather than p) of turning into a 1 at the next round.

stochastic process that at *every round* i) turns every internal 0-node into a 1-node, independently, with probability p at each node (this is because in our channel model a node has probability at most p of receiving a message), and ii) to every node i such that $i > k$ and $i - k \le r_t$ assigns probability p of turning into a 1. At a slight abuse of terminology, we shall call this process the IDEALIZED strategy, even though a transmission strategy satisfying i) and ii) may not exist for certain $S(n)$ (see Figure 3).

DEFINITION 2 (APPROXIMATION FACTOR). *Let $E(\mathbf{k}(\mathbf{t}))$ be the expected value of the random variable $\mathbf{k}(\mathbf{t})$ denoting the position of the leftmost 1-node in $S_n(t)$ under the* IDEALIZED *dissemination strategy. A certain message dissemination strategy $\mathcal{S}$ is said to be within a factor (α, β) from optimal if and only if: i) at each round gives chance at least p/α of turning into a 1 to all internal 0-nodes, and ii) $E_{\mathcal{S}}(\mathbf{k}(\mathbf{t})) \ge \frac{E(\mathbf{k}(\mathbf{t}))}{\beta}$, where $E_{\mathcal{S}}(\mathbf{k}(\mathbf{t}))$ is the analogue of $E(\mathbf{k}(\mathbf{t}))$ under strategy $\mathcal{S}$.*

Before proving the approximation bounds for GLOBAL we need the following technical lemmas:

LEMMA 1. *At any time t, there exist at most $r_T - 1$ consecutive internal 0-nodes in $S_n(t)$.*

PROOF. Consider any internal 0-node, and assume it is in position i. By definition, there exists at least one 1-node in position j, with $j > i$. Among such nodes, consider the one that became 1 (received a token) at the earliest time, $\bar{t}$, and denote it by $\bar{j}$. We must have $\bar{t} \le t - 1$, since i is an internal node at time t. But $\bar{j}$ must have received the token at time $\bar{t}$ from some node h, with $h < \bar{j}$ (because $\bar{j}$ is the earliest node to the left of i that received a token), and $h - \bar{j} \le r_t$. At time t both h and $\bar{j}$ have a token. Since $\bar{j} < i < h$, and $h - \bar{j} \le r_t$, i is in a run of at most $r_t - 1$ 0-nodes. $\square$

LEMMA 2. *Consider two random processes P_1 and P_2. Define P_1 to be the process in which a stage is composed of h rounds, with $h \ge 1$, and the probability for a node to correctly receiving the token at each stage is p. Define P_2 to be the process in which a node has probability $q \le 1 - (1-p)^{\frac{1}{h}}$ of correctly receiving the token at each round. Denote by $E_1(i)$ the event "a node has correctly received the token in at most i stages", and with $E_2(hi)$ the event "a node has correctly received the token in at most $h \cdot i$ rounds". For every $p > 0$, we have*

$$Prob(E_1(i)) \ge Prob(E_2(hi))$$

for each $i \ge 1$.

PROOF. In general, the probability that a node receives the token in t or fewer steps is $1 - (1 - \bar{p})^t$, where $\bar{p}$ is the probability of correctly receiving the token at each step. We then have to prove that

$$1 - (1-p)^i \ge 1 - (1-q)^{hi},$$

which can be rewritten as

$$(1-p)^i \le (1-q)^{hi},$$

which holds whenever $q \le 1 - (1-p)^{\frac{1}{h}}$. $\square$

The ratio between p and $\bar{q} = 1 - (1-p)^{\frac{1}{h}}$ is a decreasing function of p, whose minimum in the $(0,1]$ interval is obtained when $p = 1$ (see Figure 4). Note that the value of this ratio corresponds to the value of the constant α in the approximation bound for the GLOBAL strategy proved in the next theorem. It is interesting to observe that α decreases as communication becomes more and more reliable (increasing values of p), indicating that our proposed strategy is very close to optimal for high values of p.

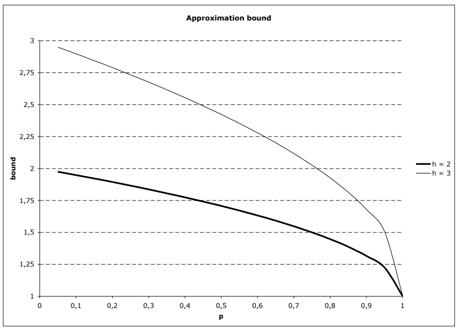

Figure 4: Approximation bound α for the GLOBAL strategy as a function of p. Plot $h = 2$ refers to the case $r_I = r_T$, plot $h = 3$ to the case $r_T < r_I \leq 2r_T$.

THEOREM 1. *Strategy* GLOBAL *is within a factor* $(\frac{p}{1-(1-p)^{\frac{1}{2}}}, 2)$ *from optimal if* $r_I = r_T$, *and within a factor* $(\frac{p}{1-(1-p)^{\frac{1}{3}}}, 3)$ *from optimal if* $r_T < r_I \leq 2r_T$.

PROOF. Due to lack of space, the proof of this theorem is omitted. □

We have observed at the beginning of this section that dissemination of emergency messages in VANETs should be fast and reliable. We can interpret these design goals under a stochastic perspective as follows. Fast backward propagation of emergency messages is related to the expected one-hop advancement of a message in the backward direction, and to the position of the leftmost transmitter in the cloud of cars. For a given value of p and r_T (and, hence, a given value of the expected one-hop message advancement), backward message propagation is sped up as much as possible by selecting the leftmost 1-node in the cloud at each round (this is part of the IDEALIZED strategy). On the other hand, reliable dissemination is maximized by giving internal 0-nodes the maximum possible chance p of turning into a 1 at each round (this is also part of the IDEALIZED strategy). These two aspects are interrelated as follows. Consider a certain node i in the network. Its chances of correctly receiving the emergency message are 0 until a node within distance r_T from it has received the message. From that point on, node i has chance at most p of turning into a 1 at each round. If we use a multiphase strategy similar to GLOBAL, by Lemma 2 this is equivalent to starting a geometric process of parameter $\bar{p} \leq p$ at node i, where the starting time t_i of the geometric process depends on the distance of the node to the initiator, and on the message dissemination strategy. Hence, the probability of node i to turn into a 1 within a certain time $\bar{t}$ is maximized when t_i is minimized, and $\bar{p} = p$. Since achieving fastest possible backward message propagation and best possible reliability at the same time is (in the general case) impossible (recall Figure 3), our goal as network designers is to find the best possible balance between reducing t_i and increasing $\bar{p}$ as much as possible.

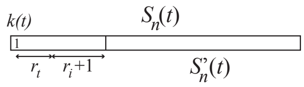

Figure 5: Notation used in the definition of the IMGLOBAL strategy.

INPUT: $S_n(t)$
OUTPUT: set T_h of transmitters for round $t+h$

$k(t)$ = leftmost 1-node in $S_n(t)$
T'_i = set of transmitters for round i computed by
GLOBAL executed on $S'_n(t)$

$T_1 = \{k(t)\} \cup T'_1$
perform all transmissions in T_1
$k(t+1)$ = leftmost 1-node in $S_n(t+1)$
$T_2 = \{k(t+1)\} \cup T'_2$
perform all transmissions in T_2

(*these instructions are needed only if* $r_T < r_I \leq 2r_T$)
$k(t+2)$ = leftmost 1-node in $S_n(t+2)$
$T_3 = \{k(t+2)\} \cup T'_3$
perform all transmissions in T_3

Figure 6: The IMGLOBAL strategy.

It is our strong feeling that, considering that p is likely to be relatively high in realistic VANET scenarios (say, $p \geq 0.75$), the dominating factor in the above described stochastic process is the starting time t_i of the geometric process. In other words, we believe performance is optimized when t_i is reduced as much as possible, and the best possible value of $\bar{p}$ is obtained after minimizing t_i.

Strategy GLOBAL is not very effective in minimizing t_i: the leftmost 1-node in the cloud is scheduled for transmission only once every 2 or 3 rounds, resulting in a backward message propagation which is a factor 2 or 3 away from optimal (see Theorem 1). In the following, we present an improved version of GLOBAL, which we call IMGLOBAL, aimed at speeding up backward message propagation as much as possible, while providing the same reliability performance as GLOBAL on all but a negligible fraction of network nodes.

Before introducing strategy IMGLOBAL we need some notation. Denote by $k(t)$ the leftmost 1-node in $S_n(t)$, and with $S'_n(t)$ the string composed of the rightmost $k(t) - r_T - r_I - 1$ bits in $S_n(t)$ (see Figure 5). Also, denote by T'_i the set of transmitters as computed by round i of GLOBAL when computed on $S'_n(t)$.

Strategy IMGLOBAL is reported in Figure 6. The strategy is very simple. Essentially, the leftmost 1-node is scheduled for transmission at each round, thus ensuring optimal backward emergency message dissemination. Reliability within a $1 - (1-p)^{\frac{1}{h}}$ factor from optimal (h is either 2 or 3 depending on r_I) is ensured on all but a negligible fraction of 0-nodes by executing GLOBAL on $S'_n(t)$. The following theorem can then easily proved:

THEOREM 2. *Algorithm* IMGLOBAL *is within a factor* $(1-(1-p)^{\frac{1}{h}}, 1)$ *from optimal, where* $h = 2$ *if* $r_I = r_T$, *and* $h = 3$ *if* $r_T < r_I \leq 2r_T$. *The bound on internal nodes is satisfied on all but at most* $c = 3r_T + r_I + 1$ *nodes, which is asymptotically negligible as* n *grows to infinity.*

PROOF. The straightforward proof of this theorem is omitted. □

Note that constant $c = 3r_T + r_I + 1$ above corresponds to the maximum possible backward advancement of the leftmost 1-node k ($3r_T$), plus a term $r_I + 1$ to avoid interference.

Concerning time complexity of our proposed dissemination strategies, the following theorem can be easily proved:

THEOREM 3. *Algorithms* GLOBAL *and* IMGLOBAL *have* $O(n)$ *time complexity.*

3.2 Estimation of time-constrained reception probability

In this section, we provide bounds to the probability $P(\bar{d}, \bar{t}, p)$ that a node at distance $\bar{d}$ from the initiator correctly receives the token within $\bar{t}$ rounds of communication. In our analysis, we consider the three dissemination strategies described in the previous section, namely IDEALIZED, GLOBAL, and IMGLOBAL.

As observed at the end of the previous section, we can provide bounds to $P(\bar{d}, \bar{t}, p)$ by bounding: *i*) the probability that a geometric process of a certain parameter $\bar{p} \leq p$ starts at position $\bar{d}$ within a certain time $\bar{t}_d \leq \bar{t}$, and *ii*) the probability of success in the geometric process within $\bar{t} - \bar{t}_d$ rounds. In the following, we denote by $Start(\bar{d}, \bar{t})$ the event "geometric process starts at node $\bar{d}$ within time $\bar{t}$ with IDEALIZED dissemination", and with $Succ(\bar{p}, h)$ the event "a geometric process of parameter $\bar{p}$ succeeds within h rounds with IDEALIZED dissemination".

In order to estimate *i*), we start by computing the expectation and variance of the 1-hop advancement of a message.

LEMMA 3. *Suppose node $k(t)$ is the leftmost 1-node in $S_n(t)$, and node $k(t)$ transmits at round $t+1$. Denote by $k(t+1)$ the leftmost 1-node in $S_n(t+1)$, and with $\mathbf{X(t)}$ the random variable $\mathbf{X(t)} = k(t+1) - k(t)$. In absence of interference, we have that*

$$E(\mathbf{X(t)}) = E(p, r_T) = r_T + 1 - \frac{1-(1-p)^{r_T+1}}{p},$$

and

$$\begin{aligned} Var(\mathbf{X(t)}) &= V(p, r_T) = \\ &= \frac{(1-p)\left[1-(2r_T+1)p(1-p)^{r_T} - (1-p)^{2r_T+1}\right]}{p^2} \end{aligned}$$

PROOF. The probability of maximal advancement (r_t positions) in one step is equal to the probability that the node at distance r_t receives the message, i.e., p. In general, the probability that the one-hop advancement is $h < r_T$, is equal to the probability that the node at distance h receives the message (p), times the probability that none of the $r_T - h$ farther nodes receives the message, i.e., $(1-p)^{(r_T-h)}$. Applying the formulas for mean ($E(\mathbf{x}) = \sum_h p_h x_h$) and variance ($Var(\mathbf{x}) = E(\mathbf{x}^2) - E(\mathbf{x})^2$) of a discrete random variable, we obtain

$$E(p, r_T) = \sum_{h=1}^{r_T} h \cdot p \cdot (1-p)^{r_T-h} = r_T + 1 - \frac{1-(1-p)^{r_T+1}}{p}.$$

and

$$\begin{aligned} V(p, r_T) &= \sum_{h=1}^{r_T} h^2 \cdot p \cdot (1-p)^{r_T-h} - E(p, r_T)^2 = \\ &= \frac{(1-p)\left[1-(2r_T+1)p(1-p)^{r_T} - (1-p)^{2r_T+1}\right]}{p^2}. \end{aligned}$$

□

Since $E(p, r_T)$ and $V(p, r_T)$ do not depend on the specific round t, we can apply well-known theorems about expectation and variance of a sum of independent, discrete random variables to prove the following lemma.

LEMMA 4. *Let $\mathbf{k(t)}$ be the random variable denoting the position of the leftmost 1-node in $S_n(t)$. We have*

$$E(\mathbf{k(t)}) = t \cdot E(p, r_T)$$

and

$$Var(\mathbf{k(t)}) = t \cdot Var(p, r_T).$$

However, Lemma 4 is scarcely useful in estimating $Prob(Start(\bar{d}, \bar{t}))$, since it gives little information on how close random variable $\mathbf{k(t)}$ is to its expected value. To circumvent this problem, we use the following concentration inequality, which is reported in [4] – Theorem 2.7, page 27:

THEOREM 4 ([4]). *Let $X_1, X_2, \ldots, X_k$ be non-negative, independent random variables. We have the following bounds for random variable $\mathbf{X} = \sum_{i=1}^{k} X_i$:*

$$Prob(\mathbf{X} \leq E(\mathbf{X}) - \lambda) \leq e^{-\frac{\lambda^2}{2\cdot\sum_{i=1}^{k} E(X_i^2)}},$$

for any $\lambda > 0$.

In our framework, the above concentration inequality can be restated as follows:

$$Prob(\mathbf{k(t)} \leq t \cdot E(p, r_T) - \lambda) \leq e^{-\frac{\lambda^2}{2\cdot t\cdot E(\mathbf{X(t)}^2)}},$$

where

$$E(\mathbf{X(t)}^2) = (r_T+1)^2 - \frac{3p + 2pr_T + (2-p)(1-p)^{r_T+1} - 2}{p^2}.$$

We can then prove the following lemma:

LEMMA 5. *Let $\bar{t}$ be such that $\bar{t} > \frac{\bar{d}-1}{E(p,r_T)}$. Then,*

$$Prob(Start(\bar{d}, \bar{t})) \geq 1 - e^{-\frac{(\bar{t}E(p,r_T)-\bar{d}+1)^2}{2\bar{t}E(\mathbf{X(t)}^2)}}.$$

PROOF. We have the following:

$$Prob(Start(\bar{d}, \bar{t})) = Prob(\mathbf{k(\bar{t})} \geq \bar{d}) = Prob(\mathbf{k(\bar{t})} > \bar{d} - 1).$$

The latter term can be rewritten as

$$Prob(\mathbf{k(\bar{t})} > \bar{t} \cdot E(p, r_T) - (\bar{t} \cdot E(p, r_T) - \bar{d} + 1).$$

The proof follows immediately by setting $\lambda = \bar{t} \cdot E(p, r_T) - \bar{d} + 1$ in the inequality of Theorem 4, and by observing that $\bar{t} > \frac{\bar{d}-1}{E(p,r_T)}$ implies $\lambda > 0$. □

The probability of event $Succ(\bar{p}, h)$ can be computed exactly, using elementary properties of the geometric process:

$$Prob(Succ(\bar{p}, h)) = 1 - (1-\bar{p})^h.$$

We now have the tools to derive lower bounds on probability $P(\bar{d}, \bar{t}, p)$ for the different dissemination strategies. We start with strategy IDEALIZED, which is easier to analyze.

Denote by $EStart(\bar{d}, \bar{t})$ the event "geometric process starts at node $\bar{d}$ exactly at time $\bar{t}$". We can write $P(\bar{d}, \bar{t}, p)$ as follows:

$$P(\bar{d}, \bar{t}, p) = \sum_{h=1}^{\bar{t}-\frac{\bar{d}}{r_T}} Prob(Succ(p, h)) \cdot Prob(EStart(\bar{d}, \bar{t} - h)).$$

In fact, in the IDEALIZED strategy an internal 0-node has probability p of turning into a 1-node at each round, once the geometric process is started. On the other hand, the starting time of the geometric process cannot be lower than $\frac{\bar{d}}{r_T}$, since the maximal backward advancement of the token at each round is r_T.

The above formula is quite difficult to handle. However, the following theorem can be used to determine, for a fixed target probability $0 < \overline{P} < 1$ and distance $\bar{d}$, an upper bound to the minimum possible value $t_{min}(\bar{d}, p)$ of $\bar{t}$ such that $P(\bar{d}, \bar{t}, p) \geq \overline{P}$.

THEOREM 5. *Given a target probability* $0 < \overline{P} < 1$ *and distance* $\bar{d}$ *to the initiator, define* $\bar{h} = \left\lceil \frac{\ln(1-\sqrt{\overline{P}})}{\ln(1-p)} \right\rceil$. *Assume emergency messages are disseminated according to strategy* IDEALIZED. *Then, we have that*

$$\min\{\bar{t} : P(\bar{d}, \bar{t}, p) \geq \overline{P}\} \leq t_{min}(\bar{d}, p) ,$$

where the value of $t_{min}(\bar{d}, p)$ *is reported in Figure 7.*

PROOF. It is easy to see that we have

$$Prob(Succ(p, h)) \geq \sqrt{\overline{P}}$$

for each $h \geq \bar{h}$.
We can now lower bound $P(\bar{d}, \bar{t}, p)$ as follows:

$$\begin{aligned} P(\bar{d}, \bar{t}, p) &= \sum_{h=1}^{\bar{h}-1} Prob(Succ(p, h)) \cdot Prob(EStart(\bar{d}, \bar{t} - h)) + \\ &+ \sum_{h=\bar{h}}^{\bar{t}-\frac{\bar{d}}{r_T}} Prob(Succ(p, h)) \cdot Prob(EStart(\bar{d}, \bar{t} - h)) \geq \\ &\geq \sqrt{\overline{P}} \cdot \sum_{h=\bar{h}}^{\bar{t}-\frac{\bar{d}}{r_T}} Prob(EStart(\bar{d}, \bar{t} - h)) = \\ &= \sqrt{\overline{P}} \cdot Prob(Start(\bar{d}, \bar{t} - \bar{h})) . \end{aligned}$$

The above inequality implies that $t_{min}(\bar{d}, p)$ equals the minimum value of $\bar{t}$ such that $Prob(Start(\bar{d}, \bar{t} - \bar{h})) \geq \sqrt{\overline{P}}$. By Lemma 5, this is equivalent to solving inequality

$$1 - e^{-\frac{((\bar{t}-\bar{h})\cdot E(p,r_T)-\bar{d}+1)^2}{2(\bar{t}-\bar{h})\cdot E(\mathbf{X}(\mathbf{t})^2)}} \geq \sqrt{\overline{P}} . \quad (1)$$

Taking the logarithm, we can transform (1) into an inequality of degree 2 in $\bar{t}$, whose minimal solution is reported in Figure 7. □

Let us now consider the GLOBAL strategy. We start with the following lemma.

LEMMA 6. *Let* $\bar{t}$ *be such that* $\lfloor \frac{\bar{t}}{k} \rfloor > \frac{\bar{d}-1}{E(p,r_T)}$, *where* $k = 2$ *if* $r_I = r_T$ *and* $k = 3$ *if* $r_T < r_I \leq 2r_T$. *Then,*

$$Prob(Start^G(\bar{d}, \bar{t})) \geq 1 - e^{-\frac{\left(\lfloor \frac{\bar{t}}{k} \rfloor E(p,r_T)-\bar{d}+1\right)^2}{2\lfloor \frac{\bar{t}}{k} \rfloor E(\mathbf{X}(\mathbf{t})^2)}} .$$

PROOF. The proof is along the same lines as the proof of Lemma 5, given the observation that the random variable $\mathbf{k}^G(\mathbf{t})$ denoting the position of the the leftmost 1-node in $S_n(t)$ after t rounds of GLOBAL dissemination has the same distribution as the random variable $\mathbf{k}\left(\lfloor \frac{\mathbf{t}}{\mathbf{k}} \rfloor\right)$. □

THEOREM 6. *Given a target probability* $0 < \overline{P} < 1$ *and distance* $\bar{d}$ *to the initiator, define* $\bar{h}^G = k \cdot \bar{h}$, *where* $k = 2$ *if* $r_I = r_T$, $k = 3$ *if* $r_T < r_I \leq 2r_T$, *and* $\bar{h}$ *is defined as in Theorem 5. Assume emergency messages are disseminated according to strategy* GLOBAL. *Then, we have that*

$$\min\{\bar{t} : P(\bar{d}, \bar{t}, p) \geq \overline{P}\} \leq t^G_{min}(\bar{d}, p) ,$$

where $t^G_{min}(\bar{d}, p)$ *is reported in Figure 7-b).*

PROOF. By Theorem 1, GLOBAL is at least as good as a strategy which is at most a factor $\frac{p}{1-(1-p)^{\frac{1}{k}}}$ from optimal for what concerns reliability. This means that, denoting with $Succ^G(p, h)$ the same event as $Succ(p, h)$ when strategy GLOBAL is used for message dissemination, we have that

$$Prob(Succ^G(p, h)) = 1 - (1 - p)^{\frac{h}{k}} .$$

To compute the minimum value $\bar{h}^G$ such that

$$Prob(Succ^G(p, \bar{h}^G)) \geq \sqrt{\overline{P}} ,$$

we observe that, in order for Lemma 2 to hold, $\bar{h}^G$ must be a multiple of k. Hence, we have

$$\bar{h}^G = \left\lceil \frac{1}{k} \cdot \frac{\ln(1 - \sqrt{\overline{P}})}{\frac{1}{k}\ln(1 - p)} \right\rceil \cdot k = k \cdot \bar{h} .$$

By proceeding as in the proof of Theorem 5, we can write

$$P(\bar{d}, \bar{t}, p) \geq \sqrt{\overline{P}} \cdot Prob(Start^G(\bar{d}, \bar{t} - \bar{h}^G)) .$$

The derivation of t_{min} can then be done by applying Lemma 6 and proceedings along the same lines as in the proof of Theorem 5. □

Note that the value of t^G_{min} reported in Figure 7-b) is obtained by omitting the floor operator in the bound stated in Lemma 6, i.e. considering the $\bar{t}/k$ ratio as a real value. Hence, what reported in Figure 7-b) is actually an approximation of t^G_{min}. However, the error committed is very small ($< k$), and asymptotically negligible.

Consider the IMGLOBAL strategy. In this case, backward message dissemination is optimal, and, similarly to strategy GLOBAL, the geometric process, once started, is at most a factor $\frac{p}{1-(1-p)^{\frac{1}{k}}}$ away from optimal (see Theorem 2) on all but a fraction at most $c = 3r_T + r_I + 1$ of internal nodes. These (at most) c nodes are those immediately to the right of the leftmost 1-node in the cloud of cars (see Figure 5). This implies that IMGLOBAL is at least as good as a strategy with optimal backward message propagation, and whose starting time of the geometric process at node i is delayed such that the leftmost 1-node in the network is at least c positions away from i. In other words, denoting with $Start^{IG}(\bar{d}, \bar{t})$ the same event as $Start(\bar{d}, \bar{t})$ when strategy IMGLOBAL is used for message dissemination, we can write:

$$Prob(Start^{IG}(\bar{d}, \bar{t})) \geq Prob(Start(\bar{d} + c, \bar{t})) .$$

By proceeding as in the proof of Theorem 6, we can then prove the following theorem:

THEOREM 7. *Given a target probability* $0 < \overline{P} < 1$ *and distance* $\bar{d}$ *to the initiator, define* $\bar{h}^{IG} = k \cdot \bar{h}$, *where* $k = 2$ *if* $r_I = r_T$, $k = 3$ *if* $r_T < r_I \leq 2r_T$, *and* $\bar{h}$ *is defined as in Theorem 5. Assume emergency messages are disseminated according to strategy* IMGLOBAL. *Then, we have that*

$$\min\{\bar{t} : P(\bar{d}, \bar{t}, p) \geq \overline{P}\} \leq t^{IG}_{min}(\bar{d}, p) ,$$

where $t^{IG}_{min}(\bar{d}, p)$ *is obtained from the formula reported in Figure 7-a) by substituting* $\bar{d}$ *with* $(\bar{d} + c)$.

4. DISCUSSION

4.1 Distance-time vs. 1-hop reliability tradeoff

Based on the bounds to t_{min} derived in the previous section, we now analyze the tradeoff between the three parameters (distance $\bar{d}$ from the initiator, reception time $\bar{t}$, and 1-hop channel reliability p) influencing $P(\bar{d}, \bar{t}, p)$. This tradeoff is analyzed for the three dissemination strategies introduced in Section 3.1. Strategy IDEALIZED is considered only for the sake of comparison.

In investigating the above mentioned tradeoff, we identify three traffic scenarios (light, medium, and heavy traffic), which differ only on the inter-vehicle distance ($60m$, $30m$, and $15m$, respectively)[3]. Independently of the traffic scenario, we

[3]These inter-vehicle distances must be intended as the composition of distances in a multi-lane road (e.g., a two lane road with $30m$ inter-vehicle distance in each lane corresponds to our high traffic scenario).

a)

$$t_{min}(\bar{d},p)=\frac{\overline{h}E(p,r_T)^2+(\bar{d}-1)E(p,r_T)-E((\mathbf{X}(\mathbf{t})^2)\ln\left(1-\sqrt{\overline{P}}\right)+\sqrt{E((\mathbf{X}(\mathbf{t})^2)\ln\left(1-\sqrt{\overline{P}}\right)\left(-2\bar{d}E(p,r_T)+2E(p,r_T)+E((\mathbf{X}(\mathbf{t})^2)\ln\left(1-\sqrt{\overline{P}}\right)\right)}}{E(p,r_T)^2}$$

b)

$$t^G_{min}(\bar{d},p)=\overline{h}+\frac{k\left((\bar{d}-1)E(p,r_T)-E(\mathbf{X}(\mathbf{t})^\mathbf{2})\ln(1-\sqrt{\overline{P}})\right)+k\sqrt{E(\mathbf{X}(\mathbf{t})^\mathbf{2})\ln(1-\sqrt{\overline{P}})(-2\bar{d}\cdot E(p,r_T)+2E(p,r_T)+E(\mathbf{X}(\mathbf{t})^\mathbf{2})\ln(1-\sqrt{\overline{P}}))}}{E(p,r_T)^2}$$

Figure 7: Value of $t_{min}(\bar{d},p)$ in the statement of Theorem 5 (a)), and in the statement of Theorem 6 (b)).

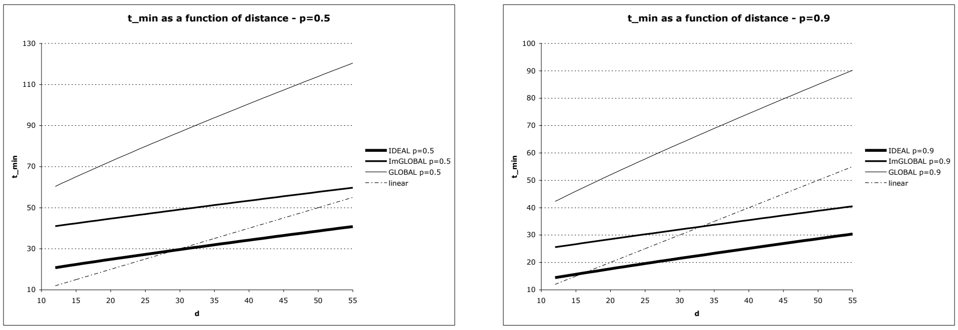

Figure 8: Values of t_{min} for fixed values of p and varying values of $\bar{d}$. Medium traffic scenario.

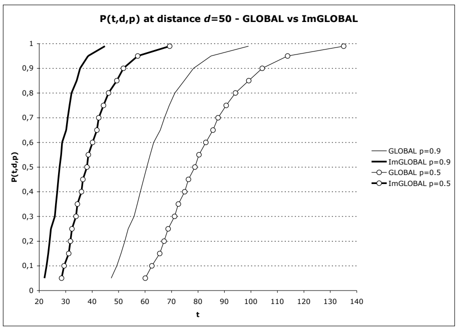

Figure 9: Values of $P(\bar{d},\bar{t},p)$ for $\bar{d}=50$ and varying values of $\bar{t}$. Medium traffic scenario.

make the following assumptions about the wireless channel model: $r_T = 120m$, which corresponds to having a normalized value of the transmit range r_T of 2, 4, and 8 under light, medium, and high traffic, respectively; and $r_I = 2r_T = 240m$.

While larger values of the transmit range are in principle possible in VANETs, increase in 1-hop channel reliability usually comes at the expense of a reduced communication range. Recent research has indicated that a 'reliable transmit range' in the order of one hundred meters is a realistic value [13]. Concerning the choice of the interference range value, assuming $r_I = 2r_T$ corresponds to a worst-case situation for our proposed message dissemination strategies.

Dependence on distance. We first investigate how (our bound on) $P(\bar{t},\bar{d},p)$ varies with distance. To this purpose, we consider medium traffic conditions ($r_T = 4$), and two different scenarios for 1-hop channel reliability: low reliability ($p = 0.5$), and high reliability ($p = 0.9$). In both scenarios, we evaluated the value of t_{min} under the different dissemination strategies as a function of distance (see Figure 8). The target probability $\overline{P}$ is set to 0.95.

As seen from the figure, t_{min} shows a sublinear increase with distance with IDEALIZED and IMGLOBAL dissemination (both in case of low and high channel reliability), and a slightly super-linear increase with GLOBAL dissemination. It is interesting to note that the slope of t_{min} expressed as a function of $\bar{d}$ displayed under IMGLOBAL dissemination is very close to the one displayed under the IDEALIZED dissemination strategy.

Regarding the effect of different reliability values of the wireless channel on the value of t_{min}, we observe the following: a considerable increase in the channel reliability (from $p = 0.5$ to $p = 0.9$), results in a percentage reduction of t_{min} which decreases with distance, and varies from 37.8% ($\bar{d} = 12$) to 32.3% ($\bar{d} = 55$) with IMGLOBAL dissemination, and from 29.9% ($\bar{d} = 12$) to 25.1% ($\bar{d} = 55$) with GLOBAL dissemination. On the other hand, given the same channel reliability, the message dissemination strategy has a considerable influence on t_{min}: at distance $\bar{d} = 55$, using IMGLOBAL dissemination reduces t_{min} of 50.39% with respect to the case of GLOBAL dissemination with low channel reliability, while the percentage reduction becomes 54.9% in case of high channel reliability.

The above results clearly indicates that designing a smart message dissemination strategy is fundamental in order to implement fast and reliable multi-hop message propagation. With respect to this point, we observe that while an increase in channel reliability typically comes at the expense of some form of resource (bandwidth) wastage, designing a smart dissemination strategy does not entail any additional resource wastage. Actually, the opposite usually holds, i.e. smart message dissemination strategies (such as IMGLOBAL) tend to use the bandwidth in a more efficient way.

Dependence on traffic conditions. The dependence of t_{min} on the traffic conditions at different distances and under different channel reliability values is reported in Table 1. The dissemination strategy is IMGLOBAL, and the value of the target reception probability $\overline{P}$ is 0.95. In the table, we

	distance	t_{min}, $p = 0.5$	t_{min}, $p = 0.9$	$\%reduction$
Heavy traffic	$600m$	$411,4ms$	$280,43ms$	$31,8\%$
	$1500m$	$525,61ms$	$382,76ms$	$27,2\%$
Medium traffic	$600m$	$441,93ms$	$281,19ms$	$36,4\%$
	$1500m$	$572,23ms$	$385ms$	$32,7\%$
Light traffic	$600m$	$502,39ms$	$283,77ms$	$43,5\%$
	$1500m$	$662,69ms$	$390,77ms$	41%

Table 1: Values of t_{min} under different traffic scenarios with target probability $\overline{P} = 0.95$ and ImGlobal dissemination strategy.

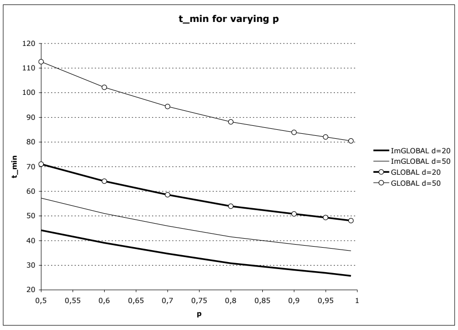

Figure 10: Values of $P(\bar{d}, \bar{t}, p)$ for fixed values of $\bar{d}$ and varying values of p. Medium traffic scenario.

assume that the size of an emergency message is 500Bytes, which is indicated as a reasonable packet size for VANETs in the security study reported in [9]. Considering that the minimum (and more reliable) data rate available in DSRC is 3Mbs, we need about $1.33ms$ to send a packet. Given this, we assume a time slot duration of $10ms$ in Table 1, which gives adequate margins for overhead due to packet header, channel access, nodes coordination, and so on.

The following observations can be made out of the data reported in Table 1. First of all, fast and reliable message dissemination appears to be a realistic goal, since t_{min} results always below $0.7s$ even at relatively long distances ($1.5Km$). Of course, what is reported in Table 1 should not be considered as reliable quantities for a real implementation, due to the many simplifying assumptions used in our analysis. Nevertheless, our findings seem to indicate that fast and reliable emergency message dissemination is indeed possible.

Another important observation regards the dependence of the relative benefits of increasing channel reliability on traffic conditions: more benefits can be observed under light traffic conditions, while the relative reduction of t_{min} with $p = 0.9$ over the case with $p = 0.5$ tends to decrease under heavy traffic conditions. This behavior seems to suggest that the relative benefit of implementing reliable 1-hop communication decreases with car density. Hence, network designers might design density-aware reliable 1-hop communication primitives, where the desired value of reliability is tuned (e.g., by appropriately setting the transmit power [12, 13], or by changing the number of MAC layer repetitions of a message [14]) depending on the observed car density[4]. It is interesting to note that setting the desired channel reliability level depending on vehicle density has the additional advantage of overcoming the well known fact that achieving a certain level of 1-hop reliability incurs higher and higher bandwidth wastage as node density increases (in fact, channel reliability is 'inversely proportional' to network congestion). Hence, by reducing the required channel reliability value in presence of high vehicle density we can considerably reduce resource wastage, while not significantly impacting fast and reliable propagation of emergency messages.

[4]Techniques for estimating vehicle density have been proposed in the literature [1].

Dependence on time. The dependence of $P(\bar{d}, \bar{t}, p)$ on time, for a fixed value of $\bar{d}$, and low and high channel reliability is reported in Figure 9. From the figure, it is clearly seen the beneficial effect of ImGlobal dissemination on the correct message reception time. It can also be observed that a higher 1-hop channel reliability induces a sharper transition (in the time domain) from near zero to close to 1 values of $P(\bar{d}, \bar{t}, p)$.

Dependence on channel reliability. The dependence of $P(\bar{d}, \bar{t}, p)$ on channel reliability, for two different values of $\bar{d}$ is reported in Figure 10. As seen from the figure, increasing 1-hop channel reliability above a certain value (about 0.9), which comes at the expense of considerable resource wastage, has only marginal effects on fast and reliable multi-hop message dissemination.

4.2 Generalizations

We believe a main contribution of this paper is the definition of a methodology that can be used to analyze multi-hop emergency message propagation under more general models. The methodology consists in subdividing reliable multi-hop message propagation into two subprocesses: *i*) backward advancement of the emergency message 'coverage area', and *ii*) reliable message propagation to the nodes which are inside the 'coverage area', but they have not received the message yet. To study *i*), one needs to estimate 1-hop message advancement, and to use this estimation to provide bounds on multi-hop message advancement. Process *ii*) can be modeled as a geometric process of a certain parameter, whose starting time is determined by *i*).

We believe the methodology used in our analysis can be used in combination with more general network topologies and/or radio channel models. For instance, one could assume that cars are not equally spaced, but spaced according to some probability distribution. It is our belief that results very similar to the ones reported in this paper hold also in a setting in which *average* node density is the same in all the network, but inter-vehicle distances can vary. In fact, 1-hop message advancement is determined by the expected number of nodes within transmitting range, and not by their relative distance; furthermore, Lemma 4 (which is used to bound multi-hop message advancement) is very general, and can be used also in this setting. Similarly, one can assume more general channel models, such as letting the probability p of correct message reception become a function of the transmitter/receiver distance. In this case, besides appropriately modeling *i*), to model *ii*) one should use a generalized notion of geometric process in which the probability of success in each experiment can change. We are currently actively working on formally deriving bounds on $P(\bar{d}, \bar{t}, p)$ under the above mentioned generalizations of our model.

5. SIMULATIONS

To assess the accuracy of the qualitative analysis reported in the previous section, we have implemented an ad hoc simulator. For the three traffic scenarios described in the previous section, the simulator disseminates emergency messages in synchronous rounds, according to the three dissem-

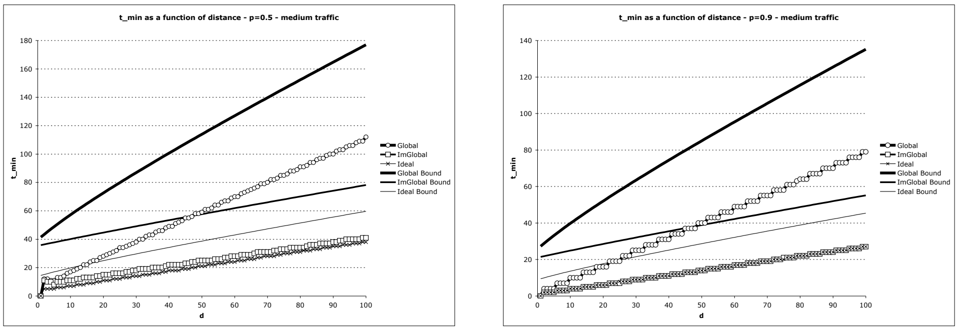

Figure 11: Values of t_{min} for fixed values of p and varying values of $\bar{d}$. Medium traffic scenario.

ination strategies (IDEALIZED, GLOBAL, and IMGLOBAL) described in Section 3.1. In each single experiments, the time (round) at which the token is first received is recorded for each car. These data are cumulated for a large set of experiments (10,000), and are used to derive the empirical distribution of $P(\bar{d}, \bar{t}, p)$, which is then compared to the bounds derived in Section 3.2.

The results of the simulations are reported in Figure 11 and **??**. The figures refer to the medium traffic scenario. Similar results, which are not reported for lack of space, have been obtained in the other traffic scenarios.

Figure 11 reports the 95% quantile of the empirical $P(\bar{d}, \bar{t}, p)$ distribution as a function of distance for the three dissemination strategies. For the sake of comparison, the figure also shows the plots of the bounds on $P(\bar{d}, \bar{t}, p)$ computed setting $\overline{P} = 0.95$. As seen from the figure, our bounds are qualitatively very accurate: the slope of the empirical distributions and that of our bounds are almost identical, independently of the 1-hop channel reliability value. It is also interesting to note that IMGLOBAL achieves virtually the same performance as IDEALIZED with high 1-hop channel reliability. This fact confirms our intuition that, when channel reliability is high, fast backward propagation of the emergency message dominates the convergence time of the geometric process on the internal 0-nodes. IMGLOBAL has optimal backward propagation, at the price of slightly delaying the start of the geometric process on a constant fraction of internal nodes. On the other hand, when p is high these internal nodes are very likely to have received the token already, thus not impairing reliable emergency message reception.

6. CONCLUSIONS

The analysis of multi-hop emergency message propagation reported in this paper has disclosed several interesting insights, such as: 1) the beneficial effect of increased 1-hop reliability tends to decrease as the distance from the emergency message initiator increases; 2) the relative benefit on multi-hop reliability of having high 1-hop reliability tends to decrease as car density increases; and, 3) the dissemination strategy has a major impact on multi-hop reliability. We believe insights 2) and 3) in particular provide very useful guidelines in the design of emergency message dissemination strategies, calling for density-aware strategies which tend to privilege fast backward propagation of the 'coverage area'. We want to outline that none of the distributed message dissemination protocols for VANETs introduced in the literature so far [3, 8, 10, 16] exploits traffic (car density) information to optimize performance. The design of a distributed dissemination strategy based on the above mentioned guidelines is matter of ongoing research.

7. REFERENCES

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