Design of Magnetic Resonance Imaging (MRI) RF Coils by Using the Method of Moments

Alessandro Rogovich\textsuperscript{1}, Agostino Monorchio\textsuperscript{1}, Paolo Nepa\textsuperscript{1}, Giuliano Manara\textsuperscript{*}, Giulio Giovannetti\textsuperscript{2}, Luigi Landini\textsuperscript{1,2}

\textsuperscript{1}Department of Information Engineering, University of Pisa, Via Caruso, I-56126 Pisa, Italy
alessandro.rogovich@iet.unipi.it

\textsuperscript{2}Institute of Clinical Physiology, Italian National Research Council (IFC-CNR)
Via G. Moruzzi 1, 56124 San Cataldo (Pisa) – Italy

Abstract – A Method of Moments (MoM) technique is employed, to design RF coils for Magnetic Resonance Imaging (MRI) applications. In particular a full wave simulator has been usefully applied to determine the principal characteristics of the antennas used for this kind of application, \textit{i.e.} the resonant modes, the $Q$ factor and the uniformity of the magnetic field radiated by the sensor. Indeed, at the increasing of the operating frequency, magneto-static models are no longer valid and more sophisticated electromagnetic tools are needed. Some examples relevant to the design of different kind of birdcage coils are presented to demonstrate the effectiveness of the method.

1. Introduction
A key issue for obtaining high quality images in Magnetic Resonance Imaging (MRI) is a proper design of the Radio Frequency (RF) coils. In particular, two basic requirements must be fulfilled: \textit{i)} in the transmission mode, RF coils must be able to produce a uniform magnetic induction field in the volume of interest so that the nuclei can be properly excited; \textit{ii)} in the receiving mode, a high signal to noise ratio (SNR) is needed, and the coil must be able to collect the signal emitted by the nuclei with a uniform amplitude throughout the volume of interest \cite{1}. If the same coil is used as both the transmitting and the receiving sensor, the above requirements are apparently related through reciprocity \cite{2}.

A properly designed RF coil must be able to provide a uniform distributed magnetic field with an appropriate level of intensity. At the same time, a high quality factor ($Q$) which is obtained by limiting the overall losses is a fundamental point to have a high SNR. Additionally, the system must operate at the resonance frequency to maximize the magnetic field strength with the applied signal.

The purpose of the paper is to show that an accurate estimation of the above mentioned design parameters is obtained by using a Method of Moments (MoM) technique. The proposed numerical method can also be successfully employed at the early stage of the design procedure for the choice of the coil configuration. The recent introduction of high magnetic fields in human studies calls for the use of frequencies up to 340 MHz; therefore a very accurate electromagnetic model of the RF coil is crucial, being the RF coil size comparable with the wavelength.

A short description of the MoM approach used for simulation is presented, and some numerical and experimental results are shown to check the validity of the proposed approach.

2. Analysis of RF coils
In the past years, the low operating frequencies used in MRI systems have allowed the design of coils by resorting to quasi-static approaches. As MRI moves towards higher and higher frequencies, RF coils have to be designed by using full-wave methods. In this context, it is important to observe that, since the coil size is usually of the order of one tenth of a wavelength, a numerical method, such as for instance the MoM, is not time consuming and
allows accurate calculations without the use of a large amount of memory. Moreover, we can take advantage of modern pre-processors with Graphic User Interface (GUI) capabilities, to analyze complex geometrical configurations.

The software developed for analyzing the RF coils is based on the MoM formulation used in the Numerical Electromagnetic Code (NEC2) solver [3]. It makes use of a hybrid formulation based on both the Electric Field Integral Equation (EFIE) and the Magnetic Field Integral Equation (MFIE) to recover the unknown current distribution on the conducting parts of the antenna. It provides an accurate modeling of a wide class of structures, as for instance thin wires or perfectly conducting smooth surfaces. Voltage sources and lumped or distributed loading can also be included in the calculations. The presence of others 3-D structures can also be accounted for, as for instance metallic shields commonly used to reduce the interaction among coils when the same coils are used to realize an MRI scanner. Metallic screen can be also used to reduce the energy radiated by the RF coil into the far field, or to realize end-caps for improving the uniformity of the field provided by the RF coil. These structures are modeled by using a patch formulation of MoM.

Furthermore, it is important to note that the frequency response of the antenna is strongly dependent on several factors that can be easily included in the numerical analysis, as for instance the lumped load capacitors used to get the required resonance frequencies, the distributed loads along the wires and the ohmic losses.

3. Numerical Results

The effectiveness of the numerical method is demonstrated by analyzing the behavior of a birdcage coil, that is a volume coil commonly used in MRI applications since it guarantees a uniform distribution of the magnetic field in its interior region and a high value of SNR [4]. Furthermore, its particular geometrical configuration allows the creation of a circularly polarized electromagnetic field rotating at the Larmor frequency, being this latter the most effective way to couple energy into the nuclei and to get a $\sqrt{2}$ factor SNR increase with respect to the linear polarization case [5].

The geometrical structure adopted for the working example is a low-pass (LP) birdcage coil made of several equispaced conductors (legs) on a cylindrical surface. Some capacitors are placed in the legs to obtain the desired resonance frequency.

In the literature, a number of different numerical techniques have been proposed for the study of a birdcage coil, as for instance the ones introduced in [2], [6] and [7], based on a magneto-static analysis of the coil. In particular, the magneto-static condition is used to recover an equivalent lumped circuit model of the coil. This simulation method is applicable only for RF coil whose size is a small fraction of the wavelength and therefore is not suited in high static induction field MRI applications. Another limiting factor of these techniques is the incomplete determination of all resonant frequencies (resonant modes). As a matter of fact, they accurately predict the correct value of the resonant frequencies for a LP birdcage coil ($N/2$ modes are present for a coil with $N$ legs), but, for the high pass configuration, they are able to predict $N/2+1$ values without taking into account the Helmholtz mode.

As a first example, we show the results relevant to an available 8 leg – LP birdcage coil, which was proposed in [7]. The coil has an height of 11cm and a diameter of 13.4 cm; the conductors are made with 1 cm wide copper strips and 2 nF capacitors. ATC 100B-American Technical Ceramics capacitors have been used because of their high quality factor. The strip conductors have been modelled by using wires with a radius of 2.23 mm computed by the equivalence formula given in [2] for ensuring an equivalent inductance when the length of the segment is the same as that of the strip.

The resonant frequencies of the structure are derived from the input admittance of the antenna; in particular, the frequency positions of the peaks of the input admittance amplitude correspond to the various resonant modes as shown in Fig. 1 (a). In Tab.1, the values derived by means of the proposed method are compared with experimental values measured on the workbench. In Tab.2, we report the frequency of the dominant mode compared with the measured value and with those obtained from the simulators described in [2] and [6]. As apparent, the MoM
solution reveals to be very accurate. The birdcage coil provides a uniform field distribution at the dominant mode resonance frequency (lowest resonant mode for a LP birdcage coil), as shown in Fig. 3. The quality factor $Q$ can be determined from the frequency response as $f_0/B$, $f_0$ being the central frequency and $B$ the -3 dB band. In Fig.2 (a) we show the $Q$ value for the simulated coil when considering the ohmic losses and ideal lossless capacitors. If the capacitor losses are taken into account the $Q$ factor shown in Fig. 2(b) is obtained. In particular, the computed value results to be $Q=501.8$, very similar to the measured one ($Q=477$) in agreement with [8]. The loss has been obtained from the capacitor datasheet [9].

As an additional example, we consider the case of a 16 leg- HP birdcage coil proposed in [10]. The resonant frequencies computed by the MoM are shown in Fig. 1 (b). The results confirm the accuracy of the technique, and the Helmholtz resonant mode is correctly predicted, whereas, the lumped circuit models proposed in [2] and [7] were not able to predict it.

### References

9. ATC 100B Series Porcelain Superchip Multilayers Capacitors Data Sheet.

### Table 1 – Resonant frequencies of the 8 legs coil under analysis.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Our method</th>
<th>Measured</th>
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<tbody>
<tr>
<td>1</td>
<td>7.8295</td>
<td>8.081</td>
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<tr>
<td>3</td>
<td>11.710</td>
<td>12.075</td>
</tr>
<tr>
<td>5</td>
<td>13.463</td>
<td>13.875</td>
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<tr>
<td>7</td>
<td>13.974</td>
<td>14.475</td>
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### Table 2 – Comparison between computed and measured resonance frequencies for the dominant mode of the 8 legs coil under investigation.

<table>
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<tbody>
<tr>
<td>Frequency (MHz)</td>
<td>7.8295</td>
<td>8.081</td>
<td>9.290</td>
<td>7.730</td>
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<tr>
<td>Relative error</td>
<td>3.12%</td>
<td>14.96%</td>
<td>4.3%</td>
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</table>
Figure 1 – Birdcage coil frequency response (a) 8 Legs Low Pass coil; (b) 16 Legs High Pass coil.

Figure 2 – Input admittance vs. frequency for the dominant mode: (a) ideal capacitor, (b) capacitor with losses.

Figure 3 – Magnetic field distribution related to the dominant mode of an 8 legs LP birdcage coil.