α - Stable Statistical Modeling and Application of Marginal Price in Electricity Market

Quan Chen[1], **Chenchi Luo**[1], **Ercan E. Kuruoglu**[2], **Zheng Yan**[1]

1. Department of Electrical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China; 2. ISTI-CNR, via G. Moruzzi 1, 56124, Italy

Abstract: In the electricity market, modeling marginal price facilitates to solve many problems in it. To model the probability density function (PDF) of system marginal price (SMP) in the spot market, positivism analysis on SMP PDF is presented in this paper. In contract to the weakness of some traditional probabilistic distribution model, an α -stable distribution model that proposed in this paper, has a better adaptation of skewness and heavy tail effect manifested by electricity prices with the theoretical support of Generalized Central Limit Theorem (GCLT). Based on this new model, an equal probability bidding strategy for generation companies is proposed in this paper. Finally, a numerical example applying the actual data from PJM electricity market, proves our deduction that α -stable distribution is more attractive, and the new bidding strategy not only increases the profit in selling electricity, but also decreases the risk in it.

Key words: electricity market; bidding strategy; probabilistic model; α -stable distribution model

1. Introduction

As one of the key elements in the electricity market, electricity price facilitates other researches in electricity market by providing the most fundamental information. The statistical distribution of electricity price is of vital importance to the analysis of such problems in the electricity market as the forecasting of the electricity price, the evaluation of market risk, the setting of bidding and energy distribution strategies, and other problems that demand the exact knowledge of probability distribution of electricity price. [1,2] However, due to the variation of generator running state, network constrains, market demand, bidding strategy of generators, and many other factors, the fluctuation in marginal prices is notable, and even price spikes are present, which increase the market risk. Therefore, many researches are focused on how to build an appropriate distribution model of marginal price.

Most literatures assumed that SMP is Gaussian distributed. [3-6] Super-Gaussian model is proposed in SMP PDF modeling [7]. When the relation between supply and demand is critical, the distribution of electricity price is far away from normal within obvious skewed peak [8]. Log normal distribution model with skewed peak is considered to solve this problem [9]. However, the above methods have their own limitations and particularities. Apparently, the Gaussian assumption cannot describe the skewness and heavy tail effect of power price distribution while other non Gaussian distributions supporting such properties are short of theoretic support. Therefore, this paper presents a general α -stable distribution model and adopts the Kolmogorov-Smirnov method for the analysis of electricity prices for a better adaptation of empirical electricity price data.

In addition, there is a lot of work on power price prediction, market risk evaluation in the application of power price probability distribution analysis. However, those on bidding strategies setting are not common. Based on the analysis of the bidding characteristics and traditional bidding methods of today's power company in the electricity market, this paper introduces a new probability distribution model of power price based on which a equal-probability classification bidding strategy on the power generation company side is proposed. Numerical examples further reveal that the proposed method not only increases the sales profit on the power generation company side, but also lowers its sales risk.

2. Statistical Distribution of Electricity Price

PJM (Pennsylvania-New Jersey-Maryland) electricity market is one of the mature electricity markets in the world. Therefore, all data in this paper comes from real-time prices of PJM market.

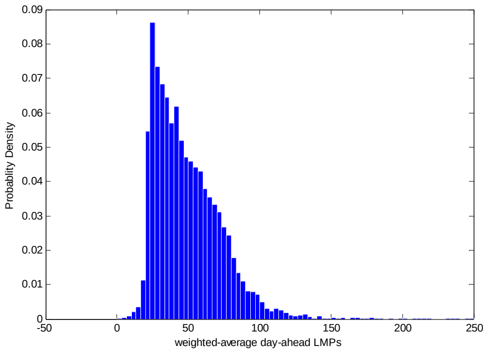

Fig. 1 Histogram of marginal prices in PJM

Observation from the histogram of marginal price shown in Fig. 1 reveals two facts:

1. Skewness

The actual histogram is highly skewed at its peak. The skewness of the distribution is much severe than normal distribution.

2. Heavy tail

Apparently, the distribution has a heavy right tail, whereas Gaussian distribution is symmetric.

Therefore, the Gaussian assumption of LMP is questionable. The universality of the Gaussian distribution is also questionable. For example, reflection off a rotating mirror yields a Cauchy distribution; hit times for a Brownian motion yields a Levy distribution. Central Limit Law (CLT) states that the sum of independent identical distributed (i.i.d.) random variables is Gaussian as the number of terms goes to infinity. However, the prerequisite of CLT is that the random variable has finite variance. However, this condition can be removed in generalized Central Limit Law which states that the limiting distribution is an α stable distribution. Actually, Gaussian, Cauchy and Levy distribution are three special cases of a more general distribution α stable distribution. α stable distribution gets its name for such a property as the summation of α stable random variables are still α stable, given that the summands all have the same α parameter.

3. α Stable Statistical Modeling of Marginal Price

3.1 The Characteristics of α Stable Distribution

Due to the lack of a closed form formula for its probability density function except for 3 special cases, namely, Gaussian, Cauchy, and Levy distributions, α stable distribution is described by its characteristic function defined as

$$\varphi(t) = E[e^{jt\mathbf{X}}] = \int_{-\infty}^{\infty} e^{jtx} dF(x) \qquad (1)$$

where $\mathbf{X} \sim S(\alpha, \beta, \sigma, \mu)$ and $F(x)$ is the cumulative distribution function . But the expression of the characteristic function of α stable distribution is not unique. According to Nolan's parameterization, the characteristic function of an α stable distribution can be expressed as[10]

$$\varphi(t) = \begin{cases} \exp\{-\sigma^{\alpha} |t|^{\alpha} [1 - j\beta sign(t) \tan\frac{\pi\alpha}{2}] + j\mu t\} & \text{if } \alpha \neq 1 \\ \exp\{-\sigma |t| [1 + j\beta \frac{2}{\pi} sign(t) \ln|t|] + j\mu t\} & \text{if } \alpha = 1 \end{cases} \qquad (2)$$

$sign(t)$ is 1 if t>0, 0 if t=0, -1 if t<0.

There are explicit meanings of each parameter in $\mathbf{X} \sim S(\alpha, \beta, \sigma, \mu)$:

$\alpha \in (0, 2]$, is referred to as tail index, determines the tail weight or the kurtosis of the distribution. As the value of α decreases, the distribution gets more peaked and exhibits heavier tails.

$\beta \in [-1,1]$, is called skewness parameter, determines the distribution's skewness. When β is positive, the distribution is skewed to the right; when β is negative, the distribution is skewed to the left; when β is zero, the distribution is symmetry about the location μ .

$\sigma \in [0, +\infty)$, is a scale parameter, determines the width of the density function.

$\mu \in \mathbb{R}$, is a location parameter, determines the shift of the peak of the density function.

Since the shape of stable distribution is determined by α and β, they are called shape parameters. Fig 2 and

Fig. 3 respectively describe the PDF with different α and β .

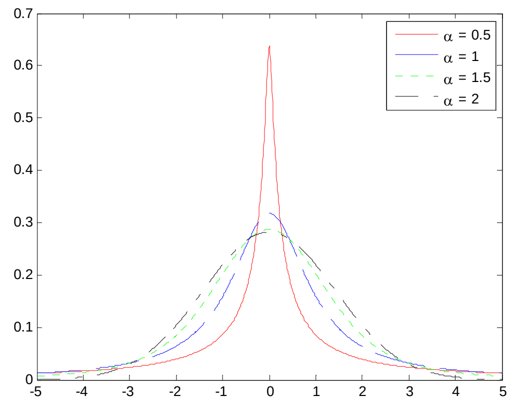

Fig. 2 PDF with different parameter α

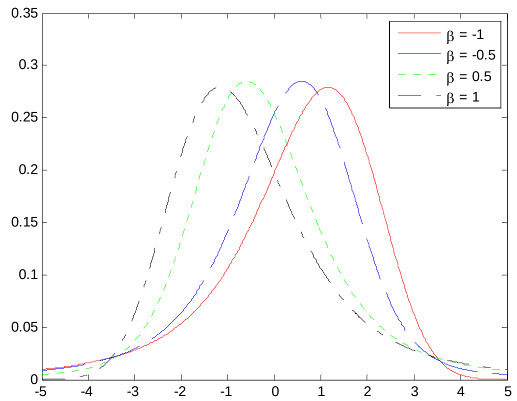

Fig 3 PDF with different parameter β

Only in 3 special case, the PDF of α stable distribution have closed form formula. Those are Gaussian ($\alpha = 2$, $\beta = 0$), Cauchy ($\alpha = 1$, $\beta = 0$) and Levy ($\alpha = 0.5$, $\beta = 1$) distributions. Fig. 4 shows the PDF of those three special α stable distributions.

There are mainly two reasons that make α stable distribution an attractive statistical tool for LMP distribution modeling. First, α stable distribution can accommodate both heavy tails and asymmetry, which enable it to give a very good fit to empirical data. Second, α stable distribution is supported by Generalized Central Limit Law (GCLT). In comparison, although student t-distribution, hyperbolic distribution and other mixtures of Gaussian distributions are all alternatives to model heavy tailed effect and skewness, they are not supported by GCLT. [11]

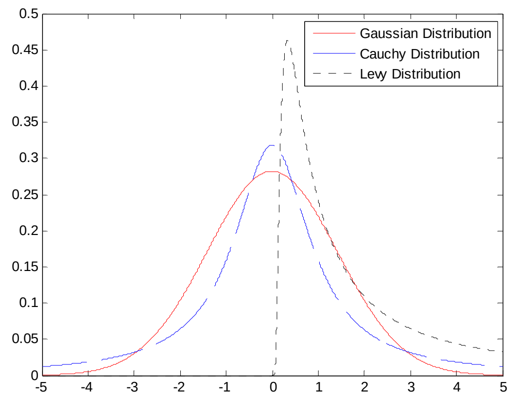

Fig. 4 The PDF of those three special α stable distributions

3.2 The parameter estimation of α stable distribution

The maximum likelihood estimate the parameter vector $\boldsymbol{\theta} = (\alpha, \beta, \sigma, \mu)$ is obtained by maximizing the log-likelihood function:

$$L_{\boldsymbol{\theta}}(x) = \sum_{i=1}^{n} f(x_i; \boldsymbol{\theta}) \qquad (3)$$

where $f(x_i; \boldsymbol{\theta})$ is the stable probability function whose explicit form is not known.

Although direct estimation methodologies can not be found for fitting these densities, numerical integration and Fast Fourier Transformation (FFT) can be applied to approximate the probability function. The maximum likelihood then undergoes a gradient searching routine for a sample of the observations to get an estimate of the parameters.

4. Generator Bidding Strategy based on α stable distribution

4.1 The Optimal Bidding Curve of Generator

As we know, the sales objective of power generation companies is to maximize the profit, that is,

Maximize: $R(q) = P \cdot q - C(q)$ (4)

s.t. $C(q) = c \cdot q^2 + b \cdot q + a$

$$q_{\min} \le q \le q_{\max} \qquad (5)$$

where, $q_{\max}$ is the summation of maximum capacity of all the on-line power generators of the power generation company, $q_{\min}$ is the corresponding minimum capacity. Therefore, the optimal bidding curve of the power generation company is,

$$P = 2 \cdot c \cdot q + b \quad \text{for} \quad q_{\min} \le q \le q_{\max} \qquad (6)$$

According to the market rule of step-wise bidding, each power generation company's bidding curve should be divided into m power segment with certain upper bounds on m which varies among different power market over the world. For example, the East China power market and PJM market of USA demand the m to be 10 maximum. But there are few papers working on the division of these m power segment. Traditionally, bidding methods are simply approximated according to $r(q)$.

The ideal optimal bidding curve of power generation companies should be a line. However, the step-wise bidding characteristics of power generation companies will make only one output point out of the m power segments reach optimal profit bidding. Other points of the segments will have certain distance from the optimal point, resulting in the loss the sales profit. The random fluctuation of power price makes the transaction capacity fall into different output capacity segments of power generators, resulting in the bidding risk (bidding means the market clear price lies in the price segment corresponding to the power segment) in each capacity segment and bring about the fluctuation risk of sales profit. It is a concise method to fathom the sales risk in term of the final profit variance. In order to effectively distribute the risks taking the advantage of the characteristics of segmental bidding strategy of power generation company side, this paper adopts a method combing clustering analysis and Monte Carlo simulation in the establishment of an optimal bidding strategy model on the power generation company side.

4.2 Equal-probability classification based bidding strategy

Due to the asymmetry of power price distribution, the bidding probability in each capacity segment differs from each other in the traditional approximating optimal segmental bidding line. We can firstly evenly divide the pricing segments according to the probabilities and take the classification center of power price as the segment center. This method brings down the sales risk on the power generation company side by making the biding probability in each capacity segment equal to each other. As Fig. 5 indicates, the pricing segment with higher bidding probability are classified in greater detail, which means that the more detailed the bidding curve are segmented the higher precision the optimal bidding can reach. For those segments with lower biding probability, the more coarsely the cluster is divided, the bolder the bidding curve will be, and correspondingly, the lower precision from the optimal bidding curve.

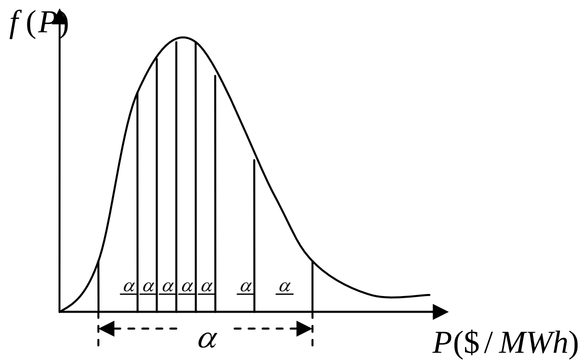

Fig. 5 Equai-probability classification of Electricity Price

Because taking the cluster center or cluster boundary is coherent in principle, the revised steps are as follows.

1. Sample the alpha stable distributed power price by Monte Carlo simulation
2. Select each cluster boundary for those price samples in the feasible segmental region on a equal-probability basis
3. For each cluster boundary power price, compute the optimal bidding output and maximum profit

4. The obtained optimal bidding output in each cluster boundaries is just the boundary of each power segment of the segmental bidding curve.

5. Numerical Examples

5.1 α Stable Statistical Modeling of Electricity Price

According to the one year (06/2006-05/2007) real price data of PJM day-ahead market, this paper compares two statistical models of marginal price – traditional normal distribution and α stable distribution proposed by this paper.

In statistics, the Kolmogorov–Smirnov test (often called the K-S test) is a goodness of fit test used to determine whether two underlying one-dimensional probability distributions differ, or whether an underlying probability distribution differs from a hypothesized distribution.

The Kolmogorov-Smirnov statistic for a given cumulative distribution function F(x) is defined by:

$$D = \sup_x \left| F(x) - \widehat{F}(x) \right| \qquad (7)$$

where $\widehat{F}(x)$ is the empirical CDF obtained from sample data.

Based on the price data of all time segments of the whole year, the results of parameter estimation of α stable distribution are $\alpha = 1.4, \beta = 1, \sigma = 12.10, \mu = 56.96$. The Kolmogorov-Smirnov statistics of the two distributions are Stable D = 0.051127, Gaussian D = 0.10792. The fitting effects of their PDF and CPF are shown in Fig. 6 and Fig. 7, respectively.

Two typical time segments are picked out to make the same comparison in the following. In which, The effects in time segment 4 are shown in Fig.8. The four parameters of α stable distribution are estimated as $\alpha = 1.28, \beta = 0.81, \sigma = 3.78, \mu = 30.76$. The Kolmogorov-Smirnov statistics of the two distributions are Stable D = 0.038524, Gaussian D = 0.10792.

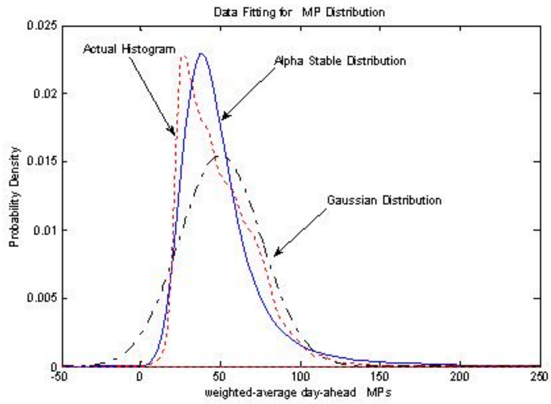

Fig. 6 The comparison of PDF between normal distribution and α stable distribution

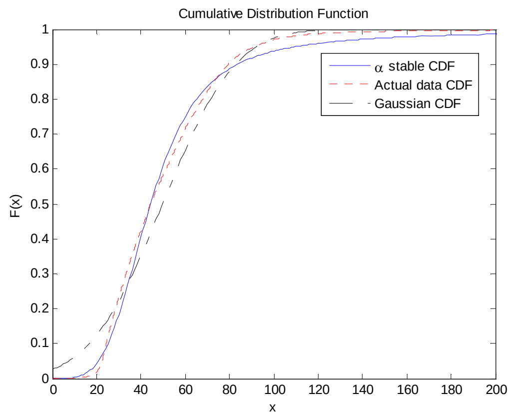

Fig. 7 The comparison of CPF between normal distribution and α stable distribution

The comparison in time segment 18 is shown in Fig.9. The four parameters of α stable distribution are estimated as $\alpha = 1.6, \beta = 1, \sigma = 13.81, \mu = 60.11$. The Kolmogorov-Smirnov statistics of the two distributions are Stable D = 0.051252, Gaussian D = 0.11721.

As it shown in Fig. 8 and Fig. 9, the statistical distribution is relative stable in load troughs (time segment 4), fitted well by α stable distribution. But due to the strong uncertainty of climate, temperature and so on, the statistical distribution of real price in load peaks (time segment 18), fluctuate more strongly. Although the fitting effect in time segment 18 is not good as in time segment 4, it is still much better than the one by normal distribution. In all cases, the Kolmogorov-Smirnov statistics satisfy

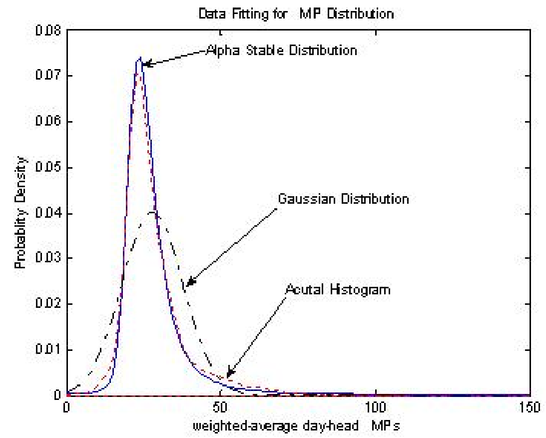

Fig. 8 The comparison of histogram with normal distribution and α stable distribution in time segment 4

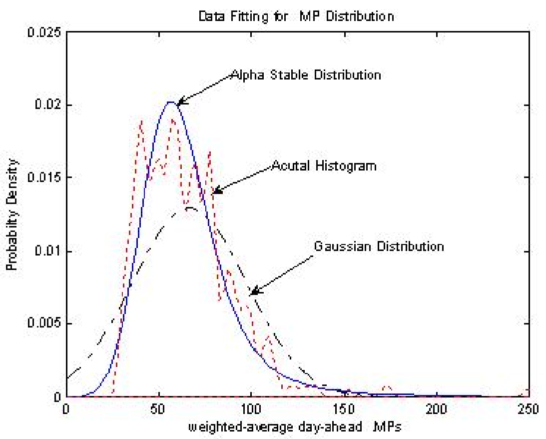

Fig. 9 The comparison of histogram with normal distribution and α stable distribution in time segment 18

Stable D <Gaussian D, which proved that α stable distribution is more suitable to approximate price statistical distribution.

5.2 Equal-probability Classification Based Bidding Strategy

Based on the α stable statistical model of marginal price, this paper realizes the equal-probability classification based bidding strategy, and tests it with time segment 4 price data of a whole year (06/2006-05/2007). The ideal linear bidding method, the traditional step-wise bidding strategy approximated by ideal bidding line, and equal-probability classification based bidding strategy proposed in this paper are compared in this section. the traditional step-wise bidding strategy mentioned here classifies the bidding segments in the available interval on average. At the point of approximation, this is the best step-wise fitting curve approximated the ideal line. The results are in the following:

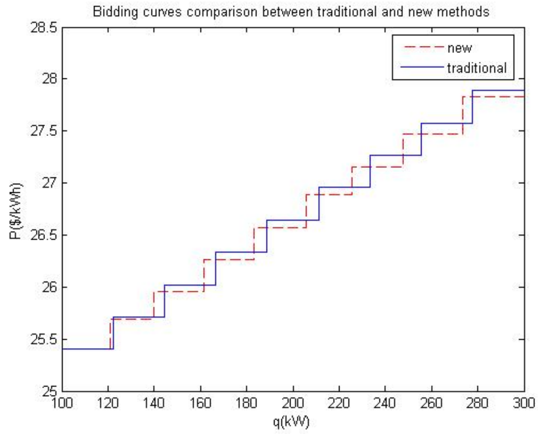

Fig. 10 The comparison of the equal-probability classification based bidding curve with the traditional step-wise bidding curve

From the comparison of the equal-probability classification based bidding with the traditional step-wise bidding strategy shown in Fig. 10, it can be found that the capacity segments are divided related compactly around 130kW and around 230kW for higher bidding probability in this interval, and coarsely divided in other interval.

Tab. 1 The comparison among ideal linear bidding, traditional step-wise bidding and equal-probability classification based bidding

	Ideal line bidding	Traditional step-wise bidding approximated by ideal line	New step-wise bidding based on equai-probability classification
Annual profit ($$10^5$)	3.9724	3.9717	3.9719
Profit variance($$10^6$)	6.5580	6.5584	6.5583

The results shown in tab. 1 indicate that, the annual profit of ideal linear bidding is the highest of the three, and the variance of profit is the least, so called ideal. But the requirement of step-wise bidding according to the market rule, make the other two bidding strategies feasible. Compared to traditional ideal line approximated bidding, not only the annual profit of equal-probability classification based bidding is higher, but also the variance of profit in that case is lower, which means the risk is lower. Therefore, the equal-probability classification based bidding strategy proposed in this paper is better than traditional ideal line approximated bidding strategy definitely.

6. Conclusions

This paper presents an α stable statistical model of marginal price for a better adaptation of skewness and heavy tail characteristics of empirical electricity price data. After analyzing the bidding characteristic in the current day-ahead electricity market, it also proposes an equal-probability classification based bidding strategy by this new model. The essence of this strategy is to close the optimal bidding solution in the high bidding probability interval. The numerical example has proved that, α stable distribution is more suitable to approximate price statistical distribution. In addition, the equal-probability classification based bidding strategy not only increases the profit in selling electricity, but also decrease the risk in it. But more factors should be considered in real bidding activities, such as whether stop generators at very low price, the constrains of generator climbing speeds and so on, which need further researches in the future work..

References

[1] VEHVILAINEN I. Basics of Electricity Derivative Pricing in Competitive Markets. Applied Mathematical Finance, 2002,9(1): 45-60

[2] VEHVILAINEN I. Applying Mathematical Finance Tools to the Competitive Nordic Electricity Market, Doctoral Dissertation. Espoo (Finland), Heisinki University of Technology, 2004

[3] ALLEN E H, ILIC M D. Stochastic Unit Commitment in a Deregulated Utility Industry. In: Proceedings of the North American Power Conference. Laramie (WY, USA): 1997.105-112

[4] REN Zhen, HUANG Fuquan, HUANG Wenying, et al. Bidding strategy for power plant in multi-electricity markets. Automation of Electric Power System, 2002, 26(2): 14-17

[5] LIU Min, WU F F. A framework for generation risk management in electricity markets. Automation of Electric Power System, 2004, 28(13): 1-6.

[6] BOTTAZZI G, SAPIO S, SECCHI A. Some Statistical Investigation on the Nature and Dynamics of Electricity Prices. Physics A, 2005, 355(1): 54-61

[7] ZHENG Hua, Xie Li, ZHANG Lizi, et al. Positivism analysis on the probability distribution of system marginal price. Proceedings of the CSEE, 2006, 26(3): 43-47

[8] ZHANG Fuqiang, ZHOU Hao. Probability distribution of prices in electricity market. Automation of Electric Power System, 2006, 30(4): 22-28

[9] ZHU Zhaoxia, Zou Bin. Statistical Analysis of Day-ahead Prices in PJM Market. Automation of Electric Power System, 2006, 30(23):53-57.

[10] Nolan, J. P. Stable Distributions: Models for Heavy Tailed Data[M]. Sp ringerVerlag, 2005.

[11] G. Samorodnitsky and M. Taqqu. Stable non-Gaussian random processes. Chapman and Hall, New York ISBN 0-412-05171-0, 1994.