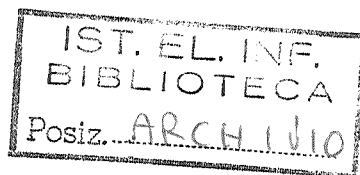


Consiglio Nazionale delle Ricerche

**ISTITUTO DI ELABORAZIONE
DELLA INFORMAZIONE**

PISA



Transformation Rules

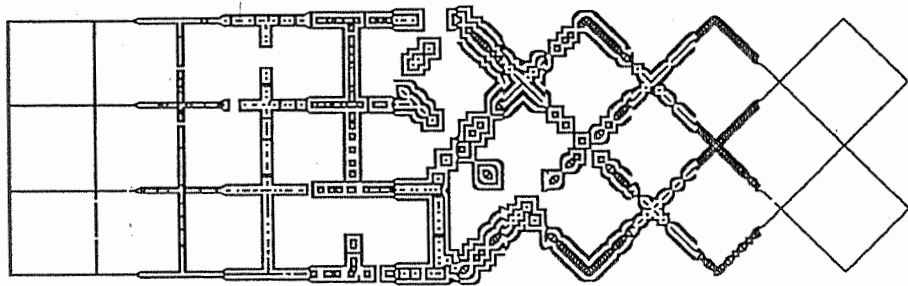
*Appendix A of "Correctness Preserving Transformation"
ESPRIT Project 2304 LOTOSPHERE
task 1.2 Second Deliverable*

R. De Nicola, A. Fantechi, S. Gnesi, P. Inverardi, M. Nesi

Nota Interna B4-26

Agosto 1991

Correctness Preserving Transformation



Edited by T. Bolognesi

Authors

Paul Boehm	TUB	Chapters 11, 12
Tommaso Bolognesi	CNUCE	Chapters 1, 3, 5, 8
Ed Brinksma	UT	Chapter 10
Ingo Classen	TUB	Chapters 11, 12
David De Frutos	UPM	Chapters 3, 7
Rocco De Nicola	IEI	Appendix A
Alessandro Fantechi	IEI	Chapter 6, Appendix A
Stefania Gnesi	IEI	Chapter 6, Appendix A
Paola Inverardi	IEI	Appendix A
Rom Langerak	UT	Chapters 2, 4, 5
Diego Latella	CNUCE	Chapters 4, 5
Elie Najm	INRIA	Chapter 9
Monica Nesi	IEI-CPR	Appendix A
Yolanda Ortega Mallen	UPM	Chapters 3, 7

Appendix A

Transformation Rules

A.1 Laws for Observational Congruence

In the following a complete set of laws for observational congruence on finite basic LOTOS is listed. This set is not minimal but includes all the laws presented in [2] while for a minimal complete set we refer to [29]. The laws for each operator are grouped together.

Choice []

$$B1 \ [] \ B2 = B2 \ [] \ B1$$

$$B1 \ [] \ (B2 \ [] \ B3) = (B1 \ [] \ B2) \ [] \ B3$$

$$B \ [] \ \text{stop} = B$$

$$B \ [] \ B = B$$

Parallel |

(the symbol | stands for any of the following operators: $||g_1, \dots, g_n||, |||, ||$)

$$B1|B2 = B2|B1$$

$$B1|(B2|B3) = (B1|B2)|B3$$

$$B1|[A]|B2 = B1|[A']|B2 \quad \text{if } A' \text{ contains the same elements of } A$$

$$B1|[A]|B2 = B1|[A']|B2 \quad A' = A \cap (L(B1) \cup L(B2))$$

$$B1|[A]|B2 = B1||B2 \quad \text{if } A \supseteq (L(B1) \cup L(B2))$$

$$B1[[]]B2 = B1|||B2$$
Enabling >>

$$stop \gg B = stop$$

$$exit \gg B = i; B$$

$$(B1 \gg B2) \gg B3 = B1 \gg (B2 \gg B3)$$

$$B \gg stop = B|||stop$$
Disabling [>

$$B1[> (B2[> B3) = (B1[> B2)[> B3$$

$$B[> stop = B$$

$$(B1[> B2)[[]]B2 = B1[> B2$$

$$stop[> B = B$$

$$exit[> B = exit[[]]B$$
Hiding hide in

$$\text{hide } A \text{ in } B = \text{hide } A' \text{ in } B \quad \text{if } A' \text{ contains the same elements of } A$$

$$\text{hide } A \text{ in } B = \text{hide } A' \text{ in } B \quad \text{if } A' = A \cap L(B)$$

$$\text{hide } A \text{ in } \text{hide } A' \text{ in } B = \text{hide } A'' \text{ in } B \quad \text{if } A'' = A \cup A'$$

$$\text{hide } A \text{ in } B = B \quad \text{if } A \cap L(B) = \emptyset$$

$$\text{hide } A \text{ in } g; B = g; (\text{hide } A \text{ in } B) \quad \text{if } \text{name}(g) \notin A$$

$$\text{hide } A \text{ in } B1 [] B2 = (\text{hide } A \text{ in } B1) [] (\text{hide } A \text{ in } B2)$$

$$\text{hide } A \text{ in } (B1 [A'] B2) = (\text{hide } A \text{ in } B1) [A'] (\text{hide } A \text{ in } B2) \quad \text{if } A \cup A' = \emptyset$$

$$\text{hide } A \text{ in } (B1 \gg B2) = (\text{hide } A \text{ in } B1) \gg (\text{hide } A \text{ in } B2)$$

$$\text{hide } A \text{ in } (B1 [> B2) = (\text{hide } A \text{ in } B1) [> (\text{hide } A \text{ in } B2)$$

Relabelling [S]

Note that in LOTOS no explicit operator for relabelling is provided, the effect of relabelling being achieved by means of the gate parameter passing in process instantiation. Therefore, for the sake of simplicity, we give the axioms by using the following notation: $[S] = [a_1/g_1, \dots, a_n/g_n]$, $S(g_i) = a_i$, $S(g) = g$ if $g \neq g_i$ for $i = 1, \dots, n$

$$\text{stop}[S] = \text{stop}$$

$$\text{exit}[S] = \text{exit}$$

$$(a; B)[S] = S(a); B[S]$$

$$(B1[]B2)[S] = B1[S][]B2[S]$$

$$(B1|[A]|B2)[S] = B1[S][|S(A)|]B2[S] \quad \text{if } S \text{ is injective on } L(B1) \cup L(B2) \cup A$$

$$(B1 >> B2)[S] = B1[S] >> B2[S]$$

$$(B1[> B2)[S] = B1[S][> B2[S]$$

$$(\text{hide } A' \text{ in } B)[S] = \text{hide } A \text{ in } B[S] \quad \text{if } S \text{ is injective on } L(B) \cup A' \text{ and } S(A') = A$$

$$B[S] = B \quad \text{if } S = \text{identity on } L(B)$$

$$B[S1] = B[S2] \quad \text{if } S1(a) = S2(a) \text{ for every } a \in L(B)$$

$$B[S1] [S2] = B[S2 \circ S1]$$

Internal Action i

$$a; i; B = a; B$$

$$B [] i; B = i; B$$

$$a; (B1 [] i; B2) [] a; B2 = a; (B1 [] i; B2)$$

Expansion Theorems

Notation: $B1[]B2[]\dots[]Bn = []\{B1, \dots, Bn\} = []S$ where $S = \{B1, \dots, Bn\}$

Hp. every element of S has the structure $b_i; B_i$.

If $B = []\{b_i; B_i | i \in I\}$ and $C = []\{c_j; C_j | j \in J\}$, then

$$B|[A]|C = []\{b_i; (B_i|[A]|C) \mid \text{name}(b_i) \notin A, i \in I\}$$

$$[] []\{c_j; (B|[A]|C_j) \mid \text{name}(c_j) \notin A, j \in J\}$$

$$\prod \prod \{a; (B_i|[A])C_j \mid a = b_i = c_j, \text{name}(a) \in A, i \in I, j \in J\}$$

$$B[> C = C \prod \prod \{b_i; (B_i[> C]) \mid i \in I\}$$

$$\text{hide } A \text{ in } B = \prod \prod \{b_i; \text{hide } A \text{ in } B_i \mid \text{name}(b_i) \notin A, i \in I\}$$

$$\prod \prod \{i; \text{hide } A \text{ in } B_i \mid \text{name}(b_i) \notin A, i \in I\}$$

$$B[S] = \prod \prod \{S(b_i); B_i[S] \mid i \in I\}$$

A.2 Recursion

In Lotos no explicit recursive operator is provided. Recursive processes can be defined by means of recursive process definitions. As we have already done for the relabelling operator, we prefer to use an explicit recursive operator to give the related laws [28].

$$\text{rec } x. E = E[\text{rec } x.E/x]$$

If $F = E[F/x]$ then $F =_{\text{rec } x.E} E$, provided that x is guarded in E (see section A.3.1)

Note that the addition of these two laws for recursive processes to the ones given before, does not provide a complete set of laws for observational congruence for basic LOTOS.

A.3 Laws for reducing unguarded recursions to guarded recursions

In the following we give the transformation rules that allow to reduce unguarded recursion for the language composed by the action prefix, (explicit) recursion, choice and pure synchronization, preserving the trace congruence.

A.3.1 Definitions

An occurrence of a process variable x in a behaviour expression E is *guarded* in E if it occurs within some subexpression $a;F$ of E , with $a \in \text{Act}$. Otherwise it is said to be *unguarded* in E [28].

A recursive process definition $\text{rec } x.E$ is said to contain a *guarded recursion* if the process variable x occurs guarded in E . Conversely, it is said to contain an *unguarded recursion* if x occurs unguarded in E .

A.3.2 Trace Congruence Laws

The following laws, in conjunction with the basic ones, form a complete set of axioms for the trace congruence for finite LOTOS.

$$i;B = B$$

$$m;(B1 [] B2) = m;B1 [] m;B2$$

A.3.3 Unguarded Recursion Laws

The following laws, in conjunction with the trace congruence and the guarded recursion ones, form a complete set of axioms for the trace congruence on the LOTOS subset composed by the action prefix, recursion, choice and pure synchronization [27].

$$\text{rec } x. (U(x)[]B(x)) = \text{rec } x. B(x)$$

$$\text{rec } x. ((U(x)[]B1(x)) \parallel B2(x)) = \text{rec } x. (B1(x) \parallel B2(x))$$

where $B(x)$, $B1(x)$, $B2(x)$ are generic behaviour expressions, and $U(x)$ is an unguarded expression of the form x or $x \parallel B(x)$

Useful instances of the above axioms are:

$$\text{rec } x. x = \text{stop}$$

$$\text{rec } x. (x [] B(x)) = \text{rec } x. B(x)$$

$$\text{rec } x. (x \parallel B(x)) = \text{stop}$$

The second of these derived laws has been introduced in [28] for the language without synchronization.

A.3.4 More Unguarded Recursion Laws

Another useful transformation, operating on a subset of LOTOS including the interleaving operator, is the following:

$$\text{rec } x. []\{a_i; Q_i\}||x = \text{rec } x. []\{a_i; Q_i||x\}$$

This law preserves strong equivalence, as presented in Chapter 6.